MAT 267: Ordinary DifferentialEquations
Problem Set 1, due October 22, 2020

1. Chapter 1 # 13
2. Chapter 2 # 4
3. By trial and error, find **one** particular solution of the ODE

   $$x' = 2x + f(t)\,,$$

   for **each** of the following right hand sides:

   (a) $f(t) = t^2$ *(try a quadratic polynomial)*;

   (b) $f(t) = e^{3t}$ *(try $x(t) = const.\, e^{3t}$)*;

   (c) $f(t) = te^{3t}$;

   (d) $f(t) = \cos t$;

   (e) $f(t) = e^{2t}$.

   Please explain your method! In any case, what is the general solution of the ODE?

4. Consider a single first-order, nonlinear ODE

   $$x' = f(x)\,,$$

   where f is continuously differentiable. Suppose that $f(0) = f(1) = 0$, and $f(x) > 0$ on $(0,1)$. Let $x(t)$ be a solution with initial value $x(0) = a \in (0,1)$. Prove the following statements.

   (a) $x'(t) > 0$ for all t, and hence x is strictly increasing;

   (b) $\lim_{t\to\infty} x(t) = 1$ and $\lim_{t\to-\infty} x(t) = 0$. In particular, $\lim_{t\to\pm\infty} x'(t) = 0$.

   (c) If $y(t)$ is a solution with initial value $y(0) = b > a$, then $y(t) > x(t)$ for all $t \in \mathbb{R}$.

   You may assume (without proof) that for each initial value the ODE has a *unique* solution.