MAT 267: Ordinary Differential Equations
Final Assessment Topics, 2021

- **Basic concepts:** Solution of an ODE; the general solution. Initial-value problems. Existence and uniqueness. How to write a higher-order equation as a first-order system

- **Scalar first-order equations**

  Slope field, solution curves

  Separation of variables. Exact equations

  Autonomous equations: Equilibria and stability. Phase portraits

  Consequences of existence and uniqueness. Examples of non-uniqueness, non-existence, and finite-time blow-up

- **Linear systems**

  General theory: The superposition principle and its consequences. The solution space of a homogeneous equation. The general solution of an inhomogeneous equation

  Eigenvalues and eigenvectors: How they give rise to particular solutions $e^{t\lambda}v$. Eigenvalues determine the dynamics ($\mathrm{Re}\,\lambda$ describes growth or decay, $\mathrm{Im}\,\lambda$ describes frequency of oscillation), while eigenvectors determine the geometry (including stable and unstable directions). Multiplicity

  Planar systems $x' = Ax$: Classification by type (saddle, node, center, spiral) and stability. Sources and sinks. Phase portraits (using eigenvalues and eigenvectors)

  Higher-dimensional systems $x' = Ax$: Diagonalization and Jordan canonical form. The general solution. How to obtain real solutions from complex ones; Matrix exponentials e^{tA}

  Duhamel's formula for solving $x' = Ax + f(t)$

  Higher order equations (constant-coefficent and Euler-Cauchy type). Mass-spring systems

- **Existence, uniqueness, and continuous dependence**

  Picard iteration. Local vs. global existence; maximal time of existence

  Consequences of existence and uniqueness

  The dynamical system $(\Phi_t)_{t\in\mathbb{R}}$ generated by a system $x' = f(x)$. The semigroup property. Vector fields and diffeomorphims

- **Linearization and stability**

  Equilibria: Nonlinear sources, sinks, and saddles; hyperbolicity. Topological conjugacy

- **Global nonlinear techniques**

  Nullclines

  Definition of *stable*, *unstable*, *asymptotically stable*

  Lyapunov functions. Gradient flows and Hamiltonian systems

  Positive and negative invariance, α- and ω-limit sets