

# MAT 267 Ordinary Differential Equations

## Tutorial 1, January 15, 2020 (Section: Almut, 10am)

### (Non-) uniqueness and lack of global existence

Consider the ODE  $x' = \frac{3}{2}x^{1/3}$ .

1. Verify that  $x \equiv 0$  solves this equation.
2. Sketch the slope field, and some solutions.
3. *Separation of Variables*  
Construct, for each initial value  $x_0 \in \mathbb{R}$ , a solution with  $x(0) = x_0$ .
4. *Non-uniqueness*  
Find infinitely many solutions with initial value  $x(0) = 0$ . Refine your sketch accordingly.
5. *Conclusion*  
What can you say when the initial value is positive? negative? What happens as  $t \rightarrow \infty$ ?  
Would you call the equilibrium  $x_0 = 0$  stable, or not?
6. *Time reversal*  
Briefly discuss the related equation  $x' = -\frac{3}{2}x^{1/3}$ .

Next consider the ODE  $x' = x^2$ .

7. Find the solution with initial value  $x(0) = x_0$ . What is its maximal interval of existence?  
(It turns out that this solution is unique (we will prove this later in the course).)
8. Sketch the slope field, the phase portrait, and a few solutions. (Consider both negative and positive initial values.)