

# MAT 267 Ordinary Differential Equations

## Tutorial 8, March 19, 2021 (Section: Almut, 10am)

### Linearization about equilibria

1. Find the equilibria of the following system.

$$x' = (2 - y)(x + y + 2)$$

$$y' = (y + 4)(x - y - 2)$$

For each of the equilibria,

- (a) determine the linearization;
- (b) classify the linearized system according to *type* (saddle, node, ...) and *stability*;
- (c) sketch a phase portrait (using all available information);
- (d) predict the behavior of the nonlinear system near the equilibrium.

Can you imagine how the nonlinear system should behave globally? Try to construct two possible scenarios.

2. *Conjugacy for sinks* (see Hirsch, Smale, & Devaney Problem 8.10). Consider a system  $x' = F(x)$  on  $\mathbb{R}^n$  near an equilibrium point  $a \in \mathbb{R}^n$ . Assume that  $F$  is continuously differentiable. Suppose that the linearization matrix  $A := DF(a)$  has  $n$  distinct negative real eigenvalues.

Describe how to construct a conjugacy between this system and the linearization  $y' = Ay$ . (Assume that  $A$  is diagonal, and use a sketch.)