Mathematicians on Creativity
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Creativity in mathematics, a vast and fascinating subject in itself, is, fortunately, not a matter of merely taking "any hypothesis that seems amusing" and deducing its consequences. If it were, it could never generate the kind of fire, despair and triumph that shine through the beautiful letters between Bolyai the elder and his son. (Mark Kac)

While Kac, like the other mathematicians quoted in this book, has no doubt about the reality of mathematical creativity, but it is not so easy to uncover what it is. Some will share the view of Paul Malliavin that mathematicians "are more interested in doing mathematics than speaking about it." However, there are few who would be as dismissive as Joseph Doob, who averred that " 'Creative process' is for the birds. I just sat around and wondered about what I was interested in."

The editors of this volume delved into the nature of creativity by assembling this compendium of quotations of mathematicians past and present on the subject. Inspired in part by a 1902 survey by two French psychologists Édouard Claparède and Théodore Flournoy and a more successful follow-up survey by Hadamard, they sent off a questionnaire to contemporary eminent mathematicians in which they asked whether their principal discoveries resulted from deliberate endeavour or arose spontaneously, how much chance, insight, inspiration or illumination played a role, how they differentiated the learning and creation of mathematics, how their practice evolved since student days and whether they analyzed the process that led to their discoveries. A followup pair of questions enquired about the *aha!* or *eureka!* experience.

As for mathematicians of the past, there was no shortage of quotations in the literature as the most famous among them were fascinated by the peculiar attributes of mathematics. Most of these older mathematicians have very little to say about creativity, but quite a bit about the qualities of mathematics, particularly the purity of its truth and beauty. Aristotle's assertion that "the mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful" sets the tone. From the modern mathematicians, quite a few themes emerge about the creative process.

Implicit in many of the quotations is that mathematics is in and of itself a superlative environment for creativity. There is ample scope for imagination and the evocation of an aesthetic response to its beauty and elegance. It presents a challenge not just to prove results, but to provide just the right context in which they are to be housed, a factor suggested by the "Proofs from the Book" spoken of by Paul Erdös. While many researchers refer to the suddenness of seeing the light towards solving solving a problem, they warn that this often follows a period of preparation and gestation. Wendell Fleming mentions that "both chance and insight are important", but that "chance will favour only those who are prepared."

Many creative mathematicians refer to their persistence and autonomy. Another aspect for many is how one's time is budgeted; they need to have some contrasting activity that will allow for a subconscious period of gestation. This often leads to a sudden sense of seeing the light. Several preferred to check known results for themselves before relying on the literature. This perhaps reflects that for many the solving of a problem is secondary to arriving at an understanding of the situation.

From my own conversations with first-rate mathematicians, I have found that many of them can canvass and evaluate a range of ideas very quickly, going over territory that I would spend much more time exploring. Combined with this is a certain ruthlessness in discarding unproductive lines of enquiry. But the essence of creativity, it seems to me, is the ability to frame a situation in a completely novel way to come up with an approach that few would think of, regardless of the time available. Another possible ingredient is the ability to focus on important problems and ideas. In the words of Poincaré, "Discovery in discernment, choice. ... The sterile combinations do not even present themselves to the mind of the creator." ([**7**]; see also page 107 of the book under review.)

This book serves the useful function of providing in one place the varied and considered views of many in the mathematical pantheon who have tried to plumb the depths of their achievements. Other authors, such as Poincaré, Hadamard, Davis and Hersh [**7, 5, 3**] have written at more length on their take on the personal side of mathematical development. Some like Aigner and Ziegler or Stein [**1, 9**] have sought to uncover attributes of creativity through the examination of particular problems. And then there are those [**3, 4, 6, 8**] whose goal is a more systematic academic study of the phenomenon. Most readers of this review, regardless of their stature in the mathematical community, will be able to relate to the sentiments expressed in this collection.

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