The birthday cake

The children had enjoyed the afternoon birthday party. The games had been played, the meal finished and the candles blown out. Now it was time for the children to share the cake. Mother had made a beautiful square cake and had iced it across the top and down the sides, and now it had to be shared among the nine children. She wanted to be completely fair about it, and ensure that each of the children had the same amount of cake *and* the same amount of icing. She also wanted the entire cake to be distributed in a single serving with each share in one piece. How does she do it?

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One might think that the method is to make cuts parallel to the sides and split the cake into nine smaller square pieces. But the child who gets the middle piece will be terribly unhappy while those getting the corner pieces will get more than their share of the icing. The mother might have wished that she had only eight children, for then she might have cut the cake from the centre to the four corners and the four midpoints of the sides. Can this be adapted to nine children? Or can you adapt the solution for a circular cake?

Actually, it turns out that there is quite a mathematical theory behind cake cutting. Suppose that we wish to slice a unifrom cake (same consistency throughout, with no icing) among several people so that each is convinced that he is getting a fair share. For two people, the process is as follows. The first person slices at a place that he feels divides the cake fairly into two. The second person then selects one of the portions, leaving the other for the slicer. The first person should be happy, as he feels that both portions are of equal worth, the second, as he feels that if the portions are different, he has selected the larger one.

Around 1960, mathematicians began to wonder whether there was a method of dividing a cake among three or more people in such a way that there are fininely many slices and each is satisfied that she has at least a fair share. In fact, one might try to impose a stronger condition that the division be *envy-free* in that each recipient not only feels she has received at least a fair share, but does not believe that anyone else's share is preferable to her own.

This has given rise to an area of mathematics with all sorts of practical applications from divorce setlements to splitting of assets and distribution of inheritances. I will not go further into this; you can google it under "Fair division". However, you might think about how you would divide a cake among three people. This problem is solved by the "Selfridge-Conway Fair-Division Procedure" which you can also google.