

A pair of geometry problems

§1. The problem of the square and triangle.

Suppose that we have a square $BCDE$, all of whose sides have length 1. Atop this square and sharing a side with it is an equilateral triangle ABE whose sides also all have length 1. The three points A , C and D do not lie on a straight line, so there is exactly one circle that passes through the three of them. What is the radius of this circle?

This is an elementary geometry problem that can cause more or less difficulty, depending on how you look at it. When I have given this to secondary students, they often come up with quite involved ways to solve it. However, with a problem like this, it is often a disadvantage to know too much. You might try to fit it into some scheme rather than simply address it on its own terms. In fact, a reader who has either not learned or forgotten his high school geometry may have an advantage.

The key is to visualize the situation in a productive way. Imagine that triangle ABE is like a piston or plunger on top of the “cylinder” $BCDE$ and let the piston drop to the bottom, so that BE comes to rest on CD and A descends to some point F . The dimensions of the triangle ABE and the same as those of the triangle FCD , so that the lengths of FC and FD are both 1. Now how far did our piston descend to take A to F ? Why, the length 1 of the sides BC and ED .

So F is a point that is distant 1 from each of A , C and D , so that it is the centre of a circle of radius 1 that passes through these three points.

§2. A radius problem.

The foregoing problem reminded Kerry Skipper of a similar find-the-radius problem that he found in the book *Games for the Super-intelligent* by James Foxx (the famous jogger who died of a heart attack in 1984). A rectangle $ABCD$ with two sides AB and AC along diameters of a circle with centre A is drawn. The distance from C to the circumference of the circle is 2 inches. The length of the diagonal BC is 8 inches. What is the radius of the circle? Foxx noted that “the trained mind is lured into concentrating on an irrelevant aspect of the problem”.

The radius of the circle is 8 inches. The two diagonals AD and BC of the rectangle have the same length, and the former is a radius of the circle. The 2 inch dimension is irrelevant.