Two problems about numbers

(1) I am standing in a theatre line. Five sixths of all the people in the line are in front of me, and one seventh of all the people are behind me. How many people are there altogether?

(2) My grandson and I have our birthdays at the same time. One year, my age was twelve times his; several years later, my age was six time his. How old was I when he was born?

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Answers: (1) 42; (2) 55.

These are standard school algebra problems. You set up an equation, and then grind out an answer. However, many such problems can be done without algebra. While this may require more ingenuity, we may get more insight into the problem.

Look at the theatre line problem. If it has a solution, then that solution must be 42. Why? Since 5/6 of the solution is a whole number, then the solution must be a multiple of 6. Similarly, it must also be a multiple of 7. So it is a multiple of 42. Since there must be only one patron left over when we take away 5/6 and a further 1/7 of the line, the total number of people must the smallest multiple of 42, namely 42 itself. This checks out.

For the grandfather-grandson situation, when the grandfather is twelve times as old as the grandson, then the grandson has been alive for one-twelfth of the the grandfather's life. In other words, the grandfather was alive for 11/12 of his life before the grandson was born, and so his age at the grandson's birth must have been a multiple of 11.

Similarly, when the grandfather is six times as old as the grandson, then he lived 5/6 of his life by the time the grandson was born, so that his age at that time must have been a multiple of 5. So the age of the grandfather at the birth of his grandson was a multiple of both 5 and 11, which leaves 55 as the only reasonable possibility.

For those who want to see how an algebraic argument goes, let x be the number of people in the theatre line and form the equation:

$$(5/6)x + 1 + (1/7)x = x$$

and then solve for x. In the case of the second problem, we might suppose that the age of the grandson is g years when the grandfather's age is $12g$ years. Then n years later, the age of the grandson is $g + n$ and the age of the grandfather is $12g + n$. If, at this time, the grandfather's age is six times that of the lad, then we get the equation: $6(g + n) = 12g + n$ so that $5n = 6g$. Since $6g$ is a multiple of 5, then so must g be a multiple of 5. Trying out the possibilies leads to $g = 5$ as the only reasonable solution; all the others make the grandfather too old. We conclude that when the grandson is 5, the grandfather is 60, so that the grandfather was 55 when the lad was born.