

Knight and Knaves

In 1978, Raymond Smullyan, a professor of mathematical logic, produced a book with the whimsical title *What is the name of this book?* This was a collection of ingenious and challenging logical puzzles that popularized **knights**, who always told the truth, and **knaves**, who always lied. The statements of such individuals had to be logically analyzed for what could be deduced from them.

For example, suppose we have two characters, Amy and Bill. Amy asserts, “Bill and I are both knights.” Bill says, “I would tell you that Amy is a knight.” What can you say about these two people? If Amy is a knight, then her statement is true and so Bill is a knight. Bill’s statement is consistent with this status, so it is possible that both individuals are knights. If Amy is a knave, then her statement is false and there is at least one knave. Let us look at Bill. He cannot be a knight. Could he be a knave? If, as a knave, he were asked whether Amy was a knight, he would lie and say that she was. But then he really would tell you that Amy was a knight, making his statement true (which is not possible for a knave). So, since Bill cannot be either a knight or a knave when Amy is a knave, we must conclude that Amy and Bill are both knights.

Here are some other puzzles about knights and knaves for you to think about.

1. I meet three people, Amy, Bill and Carol. I ask Amy how many knights are among the three of them, but I do not hear her answer. Bill tells me that Amy said there is exactly one knight among them. Carol tells me not to believe Bill; he is lying. What are Bill and Carol?

2. Leonard and Martin are sitting on the Sharbot Lake beach. I ask Leonard whether either of the two of them is a knight. Leonard responds, and I know immediately from his answer the status of the two individuals. And so, now, should you.

3. Harry tells me that all three individuals, himself (Harry), Irma and Janet are knaves. Irma then says that there is exactly one knave among them. What can you conclude?

4. Ed tells me that all of the three individuals, himself (Ed), Frank and Gail are knaves. But Frank says that there is exactly one knight among the three. What can you conclude?

If you want more, you can either get hold of Smullyan’s book, or google “Knights and knaves”.

1. If Bill were a knight, then Carol must be a knave and Amy did say there was one knight. But Amy could not say this truthfully (since then there would be at least two knights) and she could not have lied (since we know that there would be at least one and fewer than two knights). So Bill must be a knave, and Carol a knight. Amy did not say that there was exactly one knight. She could have said truthfully that there were two knights, or falsely that there were zero or three knights. So we do not know Amy's status.

2. Suppose that Leonard is a knight. Then he must respond yes, and we have no information about Martin. Suppose that Leonard is a knave. If he responds yes, then both must be knaves. If he responds no, then there is indeed at least one knight, and that must be Martin. Since I knew what the situation was from Leonard's answer, he must have said no and Leonard was a knave and Martin a knight.

3. Harry cannot truthfully call himself a knave, so Harry must be a knave and there is at least one knight among them. If Irma is a knight, then Janet must a knight as well. If Irma is a knave, then Janet must be a knight. So Harry is a knave and Janet is a knight.

4. Ed must be a knave, and there is at least one knight. If Frank is a knight, then Gail must be a knave. The assumption that Frank is a knave puts us in an impossible situation. Gail being a knight makes Frank's statement true, and Gail being a knave makes Ed's statement true. So Ed and Gail are knaves and Frank is a knight.