Neat games for two people

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There are many fine games of strategy that two people can play. Some like checkers and chess have been around for centuries; others, like *SETS* have been recently invented. Many of these require some kind of equipment, a deck of cards or a playing board and pieces. However, there is a lot of amusement to be had with a pencil and paper.

To play the game of SIM, on a plain sheet of paper, place six distinct dots. Equip two competing players with pencils of different colours, say red and green. The players play alternately and we will suppose that the player with the red pencil makes the first move. He joins any pairs of dots with a red line. The second player then joins another pair of dots with a green line. She could use one of the dots used by the red player, or could pick two completely different dots. The play continues, with each player joining with the appropriate colour a pair of dots not previously joined. Since there are fifteen pairs of dots, the game can proceed for at most fifteen moves.

However, the game stops as soon as there are three dots, each pair of which is joined by a line of the same colour. The owner of that colour is declared the winner. If the game goes the whole fifteen moves without such a triple of dots, then the game ends in a draw.

However, it turns out that a draw is impossible. Suppose that we get to the fifteenth move and no winning triple of points has been found. When the fifteenth move has been taken, each dot is joined to the five other dots by a line. Let A be one of the dots. Since there are five lines emanating from A and two colours, three of the lines must have the same colour, say red. Let us suppose that A is joined to B, C, D by red lines. If any two of B, C, D are joined by a red line, then we have a three dots (A and these two) joined by red lines. Otherwise, every pair of B, C, D must be joined by a green line and we have a winner.

The second game, invented by the English mathematical genius, John Conway, is SPROUTS. Start with at least three distinct dots on a page. Again, there are two players with alternate moves. The first player begins by joining a pair of dots and placing a new dot in the centre of the connecting segment. The second player joins another pair of dots, and puts a new dot in the centre of the connecting segment. There are two rules: (1) no dot may have more than three segments emanating from it; (2) two segments may not cross, that is, no segment may contain an interior point of another. However, it is allowed to join a dot to itself, *i.e.*, both endpoints of the segment are the same.

To give an example of the application of the first rule, suppose that we join dots A and B by a segment and put C in the middle of this segment. Then two of the segments from C have been used up (going to A and B) and no more that one further segment can be drawn from C. To clarify the second rule, no segment joining two dots can be made to pass through a dot already on the page.

The first player unable to make a move loses the game. The game cannot go on indefinitely. To see why, imagine that at the outset, each dot has three "liberties", allowances for a segment to emanate from it. The original number of liberties is three times the number of dots. Each segment drawn takes one liberty away from each of the dots it connects, and produces one liberty at the created middle dot. Thus, each move reduces the number of liberties by one. The rule that segments cannot cross means that the game will last for fewer moves than the total number of liberties at the beginning.