Logarithms and Exponentials Test Solutions.

Answers to the Test:
1. T 2. F 3. T 4. B 5. A 6. B 7. C
Solutions and Comments:

1. For all x, $\ln(e^x) = x$; this is True.

   It is a consequence of the defining property of logs:

   $$y = \log_a x \Leftrightarrow x = a^y.$$

   Of course, you have to know that ln means $\log_e$.

2. $e^a e^b = (e^a)^b$ is *not* true for all values of a and b.

   The correct equations are

   $$e^a e^b = e^{a+b} \text{ and } (e^a)^b = e^{ab}.$$

3. If $M > 0$ and $N > 0$, then

   $$\ln\left(\frac{M}{N}\right) = \ln M - \ln N;$$

   this is True. Here are some other basic properties of logs you should know, for $M, N > 0$:

   $$\ln(MN) = \ln M + \ln N; \quad \ln(M)^k = k \ln M; \quad \ln(M)^{-1} = -\ln M; \quad \ln 1 = 0.$$

4. In the equation $\log_3 x + \log_3(x-6) = 3$ both $x > 0$ and $x - 6 > 0$, since you can only take logs of positive numbers. These restrictions imply that for this problem, $x > 6$. To solve for x, use properties of logs, and keep the restriction in mind!

   $$\begin{aligned} \log_3 x + \log_3(x-6) = 3 &\Rightarrow \log_3(x(x-6)) = 3 \\ &\Rightarrow x(x-6) = 3^3 \\ &\Rightarrow x^2 - 6x - 27 = 0 \\ &\Rightarrow (x-9)(x+3) = 0 \\ &\Rightarrow x = 9, \text{ since } x > 6. \end{aligned}$$

   So the only solution to the equation is $x = 9$.

5. Let $f(x) = \ln(x^2 + 1)$. Consider the following four statements about the graph of f :

   I. It is symmetric with respect to the x-axis.

   II. It is symmetric with respect to the y-axis.

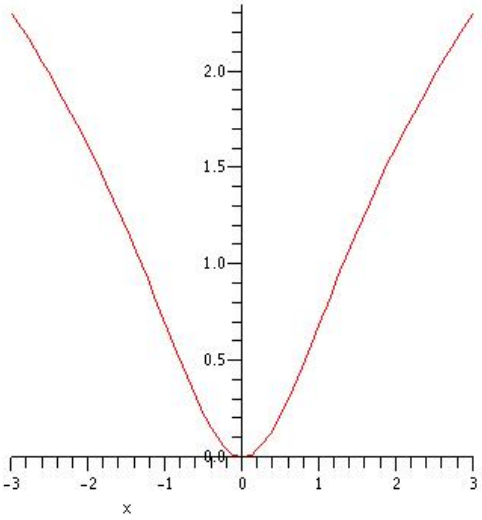
III. It is always increasing.

IV. It is always decreasing.

Only one of these statements is true, namely II. The graph *is* symmetric with respect to the y-axis:

$$f(-x) = f(x).$$

The graph is to the right. Such a graph can't be increasing (or decreasing) for all x.



6. Let $f(x) = -e^{-3x}$. Consider the following four statements about the graph of f :

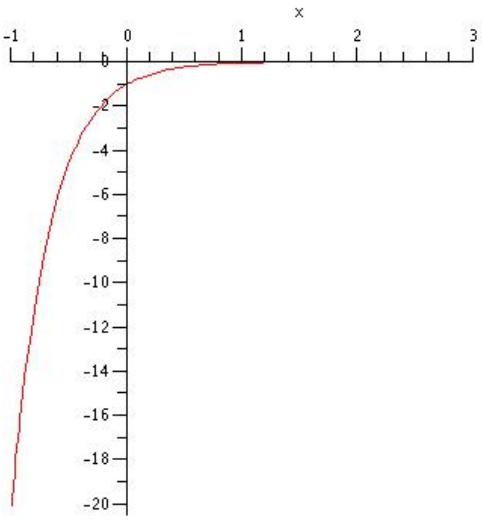
I. It is symmetric with respect to the x-axis.

II. It is asymptotic to the x-axis.

III. It is always increasing.

IV. It is always decreasing.

Only two of these statements is true, namely II and III. The graph is to the right. You can graph it by reflecting the graph $y = e^{3x}$ in the y-axis, to get the graph of $y = e^{-3x}$, and then reflecting that graph in the x-axis to get the graph of $y = -e^{-3x}$.



7. If $4^{3x-1} = 8^{3x+3}$, then $x = -\dfrac{11}{3}$.

$$4^{3x-1} = 8^{3x+3} \Rightarrow 2^{2(3x-1)} = 2^{3(3x+3)} \Rightarrow 6x - 2 = 9x + 9 \Rightarrow -3x = 11 \Rightarrow x = -\frac{11}{3}.$$