Problem 1

Solve the initial value problem for the inhomogeneous wave equation:

$$u_{tt} = u_{xx} + \cos(2\,x)\ \sin(\omega t),$$

$$u_x(0,t) = u_x(\pi,t) = 0, \qquad 0 < x < \pi,$$

$$u(x,0) = 0, \qquad u_t(x,0) = 0,$$

where ω is a real valued parameter.

Problem 2

By using Fourier Integral Formula derive the solution of the wave equation $u_{tt} = c^2\, u_{xx}, \quad x \in (-\infty,+\infty), \quad t > 0$, which satisfies the initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = 0$ when $x \in (-\infty,+\infty)$.

Transform this solution to the d'Alembert's form.

Hints for the transformation: ($\delta(x)$ is Dirac Delta function):

$$\delta(x - x_0) = \frac{1}{\pi}\int_0^{\infty} \cos(k\,(x - x_0))\,dk$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x - x_0)\,dx = f(x_0)$$

Check the answer:

$$u(x,t) = 1/\pi \int_0^{+\infty} \cos c\alpha t \int_{-\infty}^{+\infty} f(\tau)\cos(\alpha(\tau - x))d\tau d\alpha$$

Problem 3

Let F denote the periodic function, of period l, where

$$F(x) = \begin{cases} \frac{l}{4} - x, & \text{when} \quad 0 \le x \le l/2 \\ x - \frac{3\,l}{4}, & \text{when} \quad l/2 < x \le l. \end{cases}$$

[a] Describe the function $F(x)$ graphically. and show that it is, in fact, the even periodic extension, with period l, of the function

$$f(x) = \frac{l}{4} - x, \qquad 0 \le x \le l/2.$$

[b] Find the Cosine Fourier Series of the function $f(x)$.