

## MIDTERM TEST II

**Course:** APM346H **Instructor:** Chugunova Marina **Date:** 10 November 2008

*This test consists of 2 parts. Please use the backs of pages if you need more room to write. No books, notes, or calculators allowed. The total number of points is 48. GOOD LUCK!*

### Part 1. (Total 24 pts.) Quiz:

**Question 1.** (12 pts.)

Solve for  $u(x, y)$ : [a]  $u_x = 0$ , [b]  $u_{xy} = 0$ , [c]  $u_{xx} - 4u_{yy} = 0$ , [d]  $u_x - 2u_y = 0$ .

**Question 2.** (2 pts.)

Find  $du$  at the point  $(1, 2)$  if  $u(x, y) = x^2 y + y^3 x$ .

**Question 3.** (8 pts.)

$$u_{xx} - u_{yy} = 0, \quad u(x, 0) = f(x), \quad u_y(x, 0) = g(x), \quad 0 \leq x \leq 1.$$

Find all  $x$  where  $u(x, y)$  is uniquely defined if  $y$  is fixed as:

[a]  $y = 0$ , [b]  $y = 1/4$ , [c]  $y = 1/2$ , [d]  $y = 1$ .

**Question 4.** (2 pts.)

$$u_{xx} - u_{yy} = 0$$

Initial values are given on the curve  $\Gamma : \{x^2 + y^2 = 1, y > 0\}$ . At how many points on the curve  $\Gamma$  the local well-posedness theorem is not applicable? (End points  $(1, 0)$  and  $(-1, 0)$  do not belong to the curve !)

### Part 2. (Total 24 pts.)

**Problem 1.** (12 pts.)

Solve for  $u(x, y)$ . (Hint: use substitution  $u = e^{\alpha x} v$ )

$$u_{xx} - 4u_{yy} + 2u_x + u = 0, \quad u(x, 0) = x, \quad u_y(x, 0) = 0.$$

**Problem 2.** (12 pts.)

$$y u_y = u_{xx} + 9u, \quad u(0, y) = u(\pi, y) = 0, \quad y > 0.$$

[a] Solve for  $u(x, y)$  if  $u(x, 0) = 0$ . (Hint: use separation of variables  $u(x, y) = X(x)Y(y)$ )

[b] Is the initial value problem [a] well-posed in the upper half plane?  
(Hint: is solution of [a] unique ?)