Problem 1

$$x^3\, u_x - u_y = 0, \qquad u(x,0) = \frac{1}{1+x^2}$$

Characteristics are:

$$\frac{1}{2\,x^2} - y = \frac{1}{2\,\alpha^2}$$

Solution:

$$u(x,y) = \frac{1 - 2\,x^2\,y}{x^2 - 2\,x^2\,y + 1}$$

Problem 2

$$9\,u_{xx} + 6\,u_{xy} + u_{yy} + u_x = 0$$

There are infinitely many correct answers ! One of them:

$$\xi = 3\,y - x, \qquad \eta = x$$

Canonical form:

$$u_{\eta\eta} + \frac{1}{9} u_\eta - \frac{1}{9} u_\xi = 0$$

Problem 3

$$-y\,u_x + x\,u_y = u, \quad u(x,0) = f(x), \quad y > 0$$

Characteristics are:

$$y^2 + x^2 = \alpha^2$$

If $x > 0$ the solution is

$$u(x,y) = f(\sqrt{x^2+y^2})\, e^{\arcsin(\frac{y}{\sqrt{y^2+x^2}})}$$

or you can transform as

$$u(x,y) = f(\sqrt{x^2+y^2})\, e^{\arctan \frac{y}{x}}$$

If $x < 0$ the solution is

$$u(x,y) = f(-\sqrt{x^2+y^2})\, e^{\arctan \frac{y}{x}}$$