Engineering Mathematics II (2M03)
Tutorial 12

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Problem (12.3: 34)
Find *the half-range* sine and cosine expansions of the function
$f(x) = x(2-x), \quad 0 < x < 2$**.**

Solution:

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Solution:

$$f(x) = 2x - x^2$$

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$$f_{odd}(x) = \begin{cases} 2x + x^2, & -2 < x < 0 \\ 2x - x^2, & 0 < x < 2 \end{cases}$$

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We should use Fourier Sine series for f_{odd}.

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Solution:

$$f_{odd}(x) = \begin{cases} 2x + x^2, & -2 < x < 0 \\ 2x - x^2, & 0 < x < 2 \end{cases}$$

$$b_n = \frac{2}{p}\int_0^p (2x - x^2)\sin(n\frac{\pi}{p}x)dx$$

Solution:

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New variable $y = \frac{n\pi}{2}x, \quad x = \frac{2}{n\pi}y, \quad dx = \frac{2}{n\pi}dy$. **New limits of integration from** 0 **to** $n\pi$**.**

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$$= \int_0^{n\pi} \left(\frac{4}{\pi n}y - \frac{4}{\pi^2 n^2}y^2\right)\sin(y)\frac{2}{\pi n}dy =$$

Solution:

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$$= \frac{8}{\pi^2 n^2}\int_0^{n\pi} y\sin(y)dy - \frac{8}{\pi^3 n^3}\int_0^{n\pi} y^2\sin(y)dy$$

Solution:

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The formula of the sine expansion is:

$$f_{odd}(x) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}x)$$

Solution:

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Solution:

$$f_{even}(x) = \begin{cases} -2x - x^2, & -2 < x < 0 \\ 2x - x^2, & 0 < x < 2 \end{cases}$$

$$a_0 = \frac{2}{p} \int_0^p (2x - x^2)dx$$

Solution:

$$f_{even}(x) = \begin{cases} -2x - x^2, & -2 < x < 0 \\ 2x - x^2, & 0 < x < 2 \end{cases}$$

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The formula of the cosine expansion is:

$$f_{even}(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos(n\frac{\pi}{p}x)$$

Solution:

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Problem (12.3: 40)
Find the particular solution of the equation $\frac{d^2x}{dt^2} + 10x = f_{odd}(t)$
there $f(t) = 1 - t, \quad 0 < t < 2$ **use** ***periodic expansion*** **(not half range !).**

Solution:

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$$f_{odd}(t) = \begin{cases} -1-t, & -1<t<0 \\ 1-t, & 0<t<1 \end{cases}$$

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$$= -2\frac{1}{n\pi}\cos(n\pi) + 2\frac{1}{n\pi} + 2\frac{1}{n\pi}[\cos(n\pi)] = \frac{2}{n\pi}.$$

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$$= -2\frac{1}{n\pi}\cos(n\pi) + 2\frac{1}{n\pi} + 2\frac{1}{n\pi}[\cos(n\pi)] = \frac{2}{n\pi}.$$

The formula of the sine expansion is:

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t)$$

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The formula of the sine expansion is:

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t) = \sum_{n=1}^{\infty}(\frac{2}{n\pi})\sin(n\pi t)$$

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We will use the general sine series to find a particular solution of the differential equation:

$$x_p(t) = \sum_{n=1}^{\infty} B_n \sin(n\pi t)$$

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t) = \sum_{n=1}^{\infty} (\frac{2}{n\pi}) \sin(n\pi t)$$

We will use the general sine series to find a particular solution of the differential equation:

$$x_p(t) = \sum_{n=1}^{\infty} B_n \sin(n\pi t)$$

$$(x_p(t))'' = \sum_{n=1}^{\infty} (-n^2\pi^2) B_n \sin(n\frac{\pi}{2}t)$$

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t) = \sum_{n=1}^{\infty}(\frac{2}{n\pi})\sin(n\pi t)$$

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$$(x_p(t))'' = \sum_{n=1}^{\infty}(-n^2\pi^2)B_n \sin(n\frac{\pi}{2}t)$$

Substitute all above into the equation:

$$\sum_{n=1}^{\infty}(-n^2\pi^2)B_n \sin(n\pi t) + 10\sum_{n=1}^{\infty} B_n \sin(n\pi t) = \sum_{n=1}^{\infty}(\frac{2}{n\pi})\sin(n\pi t)$$

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t) = \sum_{n=1}^{\infty} (\frac{2}{n\pi}) \sin(n\pi t)$$

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Substitute all above into the equation:

$$\sum_{n=1}^{\infty} (-n^2\pi^2) B_n \sin(n\pi t) + 10 \sum_{n=1}^{\infty} B_n \sin(n\pi t) = \sum_{n=1}^{\infty} (\frac{2}{n\pi}) \sin(n\pi t)$$

We should solve for B_n

$$(-n^2\pi^2 + 10) B_n = \frac{2}{n\pi}$$

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin(n\frac{\pi}{p}t) = \sum_{n=1}^{\infty} (\frac{2}{n\pi}) \sin(n\pi t)$$

We will use the general sine series to find a particular solution of the differential equation:

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Problem (12.3: 44 modified)
Find the solution of the initial value problem $1/4\frac{d^2x}{dt^2}+12x = f_{even}(t), \quad x(0) = 1, \quad x'(0) = 0$ **there** $f(t) = 2\pi t - t^2, \quad 0 < t < 2\pi$ **using** ***half range cosine expansion.*** **(Problem is modified ! not the same as in the text book.)**

Solution:

Problem (12.3: 44 modified)
Find the solution of the initial value problem $1/4\frac{d^2x}{dt^2}+12x = f_{even}(t), \quad x(0) = 1, \quad x'(0) = 0$ **there** $f(t) = 2\pi t - t^2, \quad 0 < t < 2\pi$ **using *half range cosine expansion*. (Problem is modified ! not the same as in the text book.)**

Solution:

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Solution:

$$f(t) = 2\pi t - t^2, \quad 2\pi t \quad \textbf{is the odd term,} \quad t^2 \quad \textbf{is the even term}.$$

Problem (12.3: 44 modified)

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We should use Fourier Cosine series for f_{even}**.**

$$a_0 = \frac{2}{p}\int_0^p (2\pi t - t^2)dt$$

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$$= -8\frac{1+(-1)^n}{n^2}$$

The formula of the cosine expansion is:

$$f_{even}(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos(n\frac{\pi}{p}t) = \frac{2}{3}\pi^2 + \sum_{n=1}^{\infty}(-8\frac{1+(-1)^n}{n^2})\cos(\frac{n}{2}t)$$

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We will use the general cosine series to find a particular solution of the initial value problem:

$$x_p(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\frac{n}{2}t) \qquad (x(t))'' = \sum_{n=1}^{\infty}(-\frac{n^2}{4})A_n \cos(\frac{n}{2}t)$$

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Substitute all above into the equation:

$$\sum_{n=1}^{\infty}(-\frac{n^2}{16})A_n \cos(n\frac{\pi}{2}t) + 12A_0 + 12\sum_{n=1}^{\infty} A_n \cos(\frac{n}{2}t) = \frac{2}{3}\pi^2 + \sum_{n=1}^{\infty}(-8\frac{1+(-1)^n}{n^2})\cos(\frac{n}{2}t)$$

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$$A_n = -8\frac{1+(-1)^n}{n^2}/(-\frac{n^2}{16} + 12)$$

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Find complimentary solution of the homogeneous problem:

$$\frac{d^2 x_c}{dt^2} + 12 x_c = 0$$

$$m^2 + 12 = 0, \quad m_{1,2} = 2\sqrt{3}i, \quad x_c(t) = C_1 \cos(2\sqrt{3}t) + C_2 \sin(2\sqrt{3}t)$$

Find general solution

$$x(t) = x_c(t) + x_p(t)$$

$$x(t) = C_1 \cos(2\sqrt{3}t) + C_2 \sin(2\sqrt{3}t) + \frac{\pi^2}{18} + \sum_{n=1}^{\infty} \left[-8\frac{1+(-1)^n}{n^2} / (-\frac{n^2}{16} + 12) \right] \cos(\frac{n}{2}t)$$

Find C_1 and C_2 using

$$x(0) = 1, \quad x'(0) = 0$$

..... Write the solution of the initial value problem ....

Problem (12.4: 4)
Expand in a complex Fourier series

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Solution:

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Expand in a complex Fourier series

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Solution:

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{in\pi x/p}, \qquad c_n = \frac{1}{2p}\int_{-p}^{p} f(x) e^{-in\pi x/p} dx$$

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New variable $y = -inx, \quad x = \frac{i}{n}y, \quad dx = \frac{i}{n}dy$**.**
New limits of integration from 0 **to** $-in\pi$**.**

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$$= \frac{-1}{2\pi n^2} \int_{0}^{-in\pi} y e^y dy = \frac{-1}{2\pi n^2} \left. (y e^y - y) \right|_0^{-in\pi}$$

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$$= \frac{-1}{2\pi n^2}\int_{0}^{-in\pi} ye^y dy = \frac{-1}{2\pi n^2}\left.(ye^y - y)\right|_0^{-in\pi} = \frac{i}{2n}[(-1)^n - 1]$$

Problem (12.4: 4)

Solution:

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{in\pi x/p}, \qquad c_n = \frac{1}{2p}\int_{-p}^{p} f(x)e^{-in\pi x/p}dx$$

Rewrite for $p = \pi$

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{inx}, \qquad c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx = \frac{1}{2\pi}\int_{0}^{\pi} xe^{-inx}dx =$$

New variable $y = -inx, \quad x = \frac{i}{n}y, \quad dx = \frac{i}{n}dy$.
New limits of integration from 0 **to** $-in\pi$.

$$= \frac{-1}{2\pi n^2}\int_{0}^{-in\pi} ye^y dy = \frac{-1}{2\pi n^2}\left(ye^y - y\right)\Big|_0^{-in\pi} = \frac{i}{2n}[(-1)^n - 1]$$

$$f(x) = \sum_{n=-\infty}^{+\infty} \frac{i}{2n}[(-1)^n - 1]e^{inx}$$

Good luck with 2MO3 final exam :-) !