Math 344 Winter 2002

Problem Set 6

Section 3.3: #4, 6, 8, 12.

When using Dijkstra's algorithm display your results in a table which shows the labelling of each vertex at each step of the algorithm. For example the n-th row could give the labels at the n-th step.

Section 3.4: #4, 6, 8, 11, 16, 21, 22, 23-26, 27, 28, 33, 34, 35

In the map colouring problems first convert the map to a graph.

A. In a round-robin tournament with n contestants each pair of contestants plays exactly once (so each player plays $n-1$ matches). The problem is to schedule the matches in a minimal number of days so that each player plays at most one match per day. Restate this problem as a graph colouring problem for an appropriate graph $\mathcal{G}_n$. How many edges does this graph have? Draw $\mathcal{G}_4$. Solve the problem for $n = 4$ and 5 by colouring the **edges** of K_4 and K_5 in such a way that no two edges which share a vertex have the same colour. (This is preferrable to vertex colouring since $\mathcal{G}_5$ has quite a few edges.)

B. Prove that however one selects 55 integers between 1 and 100 there will be some two that differ by 9, some two that differ by 10, a pair that differ by 12, and a pair that differ by 13. Must there be a pair differing by 11? Prove or disprove. (Hint: divide the numbers from 1 to 100 into as many disjoint subsets $\{n, n+9\}$ as possible and use the pigeonhole principle.)