Math 344 Winter 2002

Problem Set 8

Section 4.1: 27, 28, 31, 32, 33, 34, 35, 37, 47

For #47 what you must show two things: first, that if a sequence is the Prüfer sequence of a tree $\mathcal{T}$ then the procedure in problem 33 reconstructs $\mathcal{T}$ and second that the procedure in 33, applied to **any** sequence in $\{1, \ldots, n\}^{n-2}$ always yields a tree. Prove these two things and explain why they show that Prüfer's algorithm gives a one-to-one correspondence between the set of all trees on vertices $\{1, \ldots, n\}$ and the set $\{1, \ldots, n\}^{n-2}$.

Section 4.2: 4, 9, 10,11, 15, 16, 18, 20

Section 4.3: 2, 4, 27, 33, 39

In # 27 you can use either the depth first search algorithm described in this section or the find-a-cycle-and-add-loops algorithm from an earlier assignment. The latter is probably quicker and easier for examples like this that are not too big.

Section 4.4: 26, 31, 32

Section 5.1: 17, 18, 19, 23, 30, 32

Hint for #32: To extend the rectangle by one row amounts to finding a system of distinct representatives for what sequence of n sets?

A. Draw all non-isomorphic trees with two, three, four, five and six vertices. Use a branching procedure to organize your work. (Careful: different branches can lead to isomorphic trees.) Be aware that this is a different matter than constructing all the trees on a fixed set of vertices as in §4.1 #27,28. For example there are three distinct trees on 3 vertices but they are all isomorphic.

B. If a weighted graph has an edge whose weight is less than that of any other edge must that edge be contained in **any** minimal spanning tree? Prove or disprove.

C. Use backtracking to construct a sequence in $\{1, 2, 3, 4\}^{17}$ in which no sequence of length 2 occurs twice as a pair of consecutive digits, or to show that no such sequence exists. Organize your work by always writing the smallest number possible. Explain why it is impossible to find such a sequence in $\{1, 2, 3, 4\}^{18}$.
Challenge: Is it possible for each n to find a sequence in $\{1, \ldots, n\}^{n^2+1}$ such that no sequence of length 2 occurs twice as a pair of consecutive digits?

D. Use backtracking to find all systems of distinct representatives for the sets

$$\{3,4,5\},\{1,3,5\},\{1,2,4\},\{2,3,4\}.$$