Math 344 Winter 2002

Assignment 3 solutions

Disclaimer: These are intended as sketches of solutions only. There will certainly be typos and there may also be more significant errors. If you notice any significant mistakes please send me email if you are sure it is a real error, otherwise please talk to me first.

Section 7.5

12. We take the sample space to be all collections of 4 of the 20 students. It has $C(20,4)$ points, all equally likely. There are $C(8,4)$ subsets of the freshmen so the probability is $C(8,4)/C(20,4)$.

14. I interpret this to mean no adjacent repeated letters. This is perhaps no what's intended but it makes it a little more interesting. The first letter can be any one of the 26 and each successive one must be chosen from among 25 so there are $26 \cdot 25^2$ such sequences and the probability is then $26 \cdot 25^2/26^3 = 25^2/26^2$.

15. There are 4! ways to arrange the block of s's, the block of a's the f and the r. There are 9!/(4!3!1!1!) arrangements all together (all equally likely!) so the probability is 4!4!3!1!1!/9!.

27. Here we must be careful: we can't take as our sample space the set of distributions of books as they are not equally probable. For example it is less likely for Sheila to get all 9 books than for each to get 3. The sample space should be $\{R,S,T\}^9$. We want the sequences which contain 2 R's, 4 S's and 3 T's. There are $\frac{9!}{2!4!3!}$ such sequences so the probability is $\frac{9!}{2!4!3!3^9}$.

34. The low card in a straight must be between 2 and 10 so there are 36 choices for the low card and than 4 choices for each of the remaining 4 cards, so the probability is $36 \cdot 4^4/C(52,5)$.

35. Here the sample space consists of 5-element subsets of $\{1,\ldots 25\}$. A five element subset containing no consecutive numbers amounts to a sequence of 5 1's and 20 0's, the positions of the ones specifying which files are chosen, with no adjacent 1's. To get such a sequence write down 101010101 and then distribute the 16 remaining 0's among the 6 available slots in $C(16+5,5)$ ways. Answer $= C(21,5)/C(25,5)$.

26. Like #25 we want sequences of 6 1's and 48 0's containing 3 adjacent 1's and no other adjacent 1's. First write down 3 1's and one 111 separated by 0's in one of 4 ways. Then there remain 45 0's to be distributed in 5 slots in one of $C(45+4,4)$ ways. Answer $= 4C(45+4,4)/C(54,6)$.

Extra problems

1. The ball chosen is one of 3 red balls, with equal probabilities. For 2 of these balls the other ball in the urn is red so the desired probability is 2/3.

2. Using 0 as a place holder we can put either 10^r2 or 20^r1 in one of $n-(r+2)+1$ positions and then fill in in $(n-2)!$ ways. Answer $= 2(n-r-1)(n-2)!/n! = 2(n-r-1)/n(n-1)$.