Waves Generated by Airflow over a Mountain Ridge

When density-stratified air is forced by winds over elevated terrain, the vertical displacement of the flow results in a downstream pattern of dispersing waves. The effects of these gravity waves can often be visualized through clouds which do not drift with the winds, but remain stationary with respect to the topography. Such vertical disturbances to the flow are important in the understanding of the micro-scale variations in cloud and precipitation patterns for alpine communities (and ski resorts). Intense wave activity is also an aviation hazard when encountered as in-flight turbulence.

The first breakthrough in the theory of nonlinear topographic waves occurred as early as 1953. In a special case of two-dimensional, stratified flow, R. Long made the remarkable observation that the steady, nonlinear streamfunction satisfies the linear Helmholtz equation. The topographic problem is distinguished from the classical contexts of electromagnetics and acoustics through an unusual radiation boundary condition in the far-field. Although Long's theory has been extensively studied within the atmospheric sciences literature, some interesting mathematical aspects remain unexplored. A Fourier-based integral equation is shown to be surprisingly successful in computing flows despite its first-kind nature. A more robust second-kind integral equation, based on a Greens function analysis of Lyra (1943), shows promise of extending the parameter region where large amplitude solutions to Long's theory can be computed. Finally, a few glimpses of the obstacles and advances are shared from our experiences in current efforts to address the stability of these flow solutions for complex topography.

