MAT1000HF Fall 2017
Assignment 1
Due Sept 20th

Problem 1
Let A be an index set, $\{X_\alpha\}_{\alpha\in A}$ a family of non-empty sets and for each $\alpha \in A$ let $\mathcal{M}_\alpha$ be a σ-algebra on X_α. Consider the product space

$$X = \prod_{\alpha\in A} X_\alpha.$$

Let $\mathcal{M}$ be the σ-algebra generated by the cylinder sets $\{\pi_\alpha^{-1}(E_\alpha)|E_\alpha \in \mathcal{M}_\alpha, \alpha \in A\}$ and $\mathcal{M}^*$ be the one generated by boxes $\{\prod_{\alpha\in A} E_\alpha|E_\alpha \in \mathcal{M}_\alpha\}$. Show that $\mathcal{M} \subset \mathcal{M}^*$ but in general $\mathcal{M} \neq \mathcal{M}^*$

Hint 1: Proposition 1.3 implies that if A is countable then $\mathcal{M} = \mathcal{M}^*$; we should thus take A to be not countable.)
Hint 2: You might find useful to first prove the following intermediate result. For any $A' \subset A$ let $\mathcal{M}_{A'} = \mathcal{M}(\{\pi_\alpha^{-1}(E_\alpha)|E_\alpha \in \mathcal{M}_\alpha, \alpha \in A'\})$; let now

$$\tilde{\mathcal{M}} = \bigcup_{A'\subset A \text{countable}} \mathcal{M}_{A'}$$

Then show that $\mathcal{M} = \tilde{\mathcal{M}}$ (Hint^2: show that $\tilde{\mathcal{M}}$ is a σ-algebra which contains the cylinders...) The above can be loosely stated as "any set in $\mathcal{M}$ is determined by countably many coordinates"

Solve problems 2 4 5 s.t. 7 9 10 from Folland Chapter 1