

Stationary phase.

WARM UP

$$I(\tau) = \int_{\mathbb{R}^n} e^{i\tau \phi(z)} a(z) dz$$



a nearly const.

• ϕ smooth function of \mathbb{R}^n , \mathbb{R} -valued.

• $a \in C_0^\infty(\mathbb{R}^n)$ amplitude

First notice:

$$|I(\tau)| = \left| \int_{\mathbb{R}^n} e^{i\tau \phi(z)} a(z) dz \right|$$

$$\leq \int_{\mathbb{R}^n} |e^{i\tau \phi}| |a(z)| dz$$

$$\leq \|a\|_{L^1} < \infty$$

• Example - $\phi(x) = c$

$$I(\tau) = \int e^{2\pi i \tau \phi} a(z) dz$$

$$= e^{2\pi i \tau c} \int a(z) dz$$

Shows no decay at ∞ .

$$= A \cdot e^{2\pi i \tau c}$$

= let $\phi(x) = x_\ell$ $\ell = 1, \dots, n$.

$$I(\tau) = \int e^{2\pi i \tau z_\ell} a(z) dz$$

