MAT 247
ASSIGNMENT 2
DUE THURSDAY JANUARY 27

(1) (Axler 6.18) Let $V, \langle , \rangle$ be an inner product space and let $P : V \to V$ be a linear operator. Suppose that $P^2 = P$ and that $||Pv|| \leq ||v||$ for all $v \in V$. Prove that P is an orthogonal projection.

(2) (Axler 6.26) Let $V, \langle , \rangle$ be an inner product space and fix $v \in V$. Define a linear map $T : V \to \mathbb{F}$ by $T(w) = \langle v, w \rangle$. Find $T^*(a)$ for $a \in \mathbb{F}$.

(3) (Axler 6.28) Let $V, \langle , \rangle$ be an inner product space, let $T : V \to V$ be a linear operator and let $\lambda \in \mathbb{F}$. Prove that λ is an eigenvalue of T if and only if $\bar{\lambda}$ is an eigenvalue of T^*.

(4) (Axler 6.32) Suppose that A is an $n \times m$ real matrix. Prove that the dimension of the span of the rows of A equals the dimension of the span of the columns of A.