MAT 247
ASSIGNMENT 5
DUE THURSDAY MARCH 3

(1) (Axler 7.19) Suppose that T is a positive operator on an inner product space V. Prove that T is invertible if and only if
$$\langle Tv, v \rangle > 0$$
for all $v \in V \setminus \{0\}$.

(2) (Axler 7.20) Prove or disprove: the identity operator on $\mathbb{R}^2$ has infinitely many square roots.

(3) (Axler 7.23) Define a linear operator T on $\mathbb{R}^3$ by $T(a_1, a_2, a_3) = (a_3, 2a_1, 3a_2)$. Find an isometry S such that $T = S\sqrt{T^*T}$.

(4) (Axler 7.29) Let T be a linear operator an inner product space V. Prove that T is invertible if and only if 0 is not a singular value of T.