MAT 247
ASSIGNMENT 7
DUE THURSDAY MARCH 17

(1) Consider the polynomial $p(x, y) = 2x^2 + 4xy + 3y^2$.
   (a) Find a symmetric bilinear form H on $\mathbb{R}^2$ whose associated quadratic form is $p(x, y)$.
   (b) Find a basis for $\mathbb{R}^2$ for which this form is diagonal.
   (c) Use (b) to decide whether the graph of $2x^2 + 4xy + 3y^2 = 1$ is an ellipse or a hyperbola.

(2) Let V be a vector space. Let $v_1, \dots, v_n$ be a basis for V and let $\alpha_1, \dots, \alpha_n$ be the corresponding dual basis for V^*. Let $w_1, \dots, w_n$ be another basis for V and let $\beta_1, \dots, \beta_n$ be the corresponding dual basis.
   (a) Let Q be the change of basis matrix between $v_1, \dots, v_n$ and $w_1, \dots, w_n$. Show that Q^t is the change of basis matrix between $\beta_1, \dots, \beta_n$ and $\alpha_1, \dots, \alpha_n$.
   (Here by change of basis matrix, I mean that $w_j = \sum_{i=1}^{n} Q_{ij} v_i$.)
   (b) Let H be a bilinear form on V. Use (a) to prove that

$$[H]_{w_1,\dots,w_n} = Q^t \, [H]_{v_1,\dots,v_n} \, Q$$

(3) Let $V, \langle , \rangle$ be a real inner product space. Let $T : V \to V$ be a linear operator. Let $H : V \times V \to \mathbb{F}$ be the map defined by $H(v, w) = \langle v, Tw \rangle$.
   (a) Show that H is a bilinear form on V.
   (b) Show that H is symmetric if and only if T is self-adjoint.
   (c) Show that H is an inner product if and only if T is positive and invertible.