Using These Notes

Perhaps, when you learned algebra, at the very beginning you weren't sure what the point was. The very first algebra word problems ("What number do you need to add to 7 to get 15?") were so easy to do that it actually took *more* effort to translate them into the new symbology of algebra and solve them that way. But after a while you got used to the notation and rules of algebra ... and after a longer while, you started seeing problems ("Find the points where this circle intersects with this parabola") that would be extremely hard to solve from scratch, but which surrendered quickly to an algebraic approach. The investment in time to learn algebra paid off later, allowing you to do a series of nontrivial steps of mathematical reasoning without having to think about *why* every single step was valid.

The same is true of proof-based mathematics. It has a downside, especially upon first encounter: one practically needs to learn a new language and translate ideas into that language, when it seems that just dealing with the ideas directly would be easier. But proof-based mathematics has the same upside: after some time on the learning curve, it begins to come naturally, and soon we find ourselves tackling problems that would be far too complicated to solve from scratch. The time we spent thinking carefully about every little fact at the beginning pays off later, when we can just use those facts without having to justify them anew each time.

In these notes, we introduce the study of number theory from the ground up. A lot of the material, especially near the beginning, might seem so familiar or obvious that there's no point in proving it—but at least this way we'll get used to formal proofs (it can take a little exposure to catch on to what mathematicians consider a complete, rigorous proof). Later, these early facts will be among our most valuable tools for proving more complicated statements about numbers.

The material in these notes is gathered for the most part into Facts, Theorems (really important or interesting facts), and Consequences (facts that become evident after we've proved some other fact). The Theorems are the highlights of the material (no flash photography, please), both for their historical significance in the development of number theory and for their pure interest value. The various Facts have several purposes at once: they furnish exercises for developing our proof skills, they record truths or rules that we'll need later on to prove more complicated things, and of course they themselves often say interesting things about numbers. At the end of the notes are a Glossary (giving page numbers where terms were defined in bold letters throughout the notes, as well as definitions of other terms that appeared in the discussion and brief bibliographic sketches of a few notable mathematicians connected to number theory) and a Suggested Reading List; both of these are intended as references, not as something to be memorized or exhaustively studied.

If this is one of your first encounters with proof-based mathematics, *it is possible to read these notes, skipping some of the proofs.* Any given Fact, Theorem, etc. will speak for itself, and can be taken on faith and used later on. The philosophy of these notes is: all of the details are included—even more than can be assimilated the first time through—so they'll be there if you ever want to go back over them with a super rigorous eye. The goal of this SOAR program is to become exposed to the study of number theory, and to become more exposed to proof-based mathematics (but not to die of exposure!).

It's amazing that, just confining ourselves to studying objects as simple as the numbers themselves, we will be able to discover and marvel at some of the most beautiful patterns in all of mathematics. We hope to convey some of that beauty and mystery to you during the three weeks of SOAR 2000.