

MAT199H1S: Aha! Mathematical Discovery and Creative Problem Solving
Winter 2021
University of Toronto

I. Instructor and Teaching Assistants

Course Instructor: Robert McCann

Email: mccann@math.utoronto.ca (all emails should include the subject heading 199)

Office Hours: 17h10-18h00 Mondays on Zoom Course Website: www.math.toronto.edu/mccann/199

TA: Cameron Davies

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Office Hours: TBD on Zoom

II. Course Overview

Course Description

This course is an exploration into the creative process and use of imagination as they arise in the context of mathematical problem solving. The problems, which are all at a pre-calculus level, are chosen primarily by the criterion of aesthetic appeal, and emphasize reasoning rather than technique. Still, many of them are quite challenging, and substantial independent thinking will be required, the course is therefore appropriate for students from a variety of backgrounds and disciplines, including hard sciences. Its goal will be to hone each participant's creativity and mathematical problem-solving skills while guiding them towards the 'Aha!' experience which accompanies independent discovery. Restricted to first-year students. Not eligible for CR/NCR option.

Prerequisites

High school level algebra

Course Objectives

To hone each participant's creativity and mathematical problem-solving skills.

To guide them towards the 'Aha!' experience which accompanies independent discovery.

To increase their familiarity and comfort with mathematical concepts, ideas, and thinking.

To teach participants to recognize situations in which a mathematical approach may provide insights.

To improve their ability to identify and verify (or falsify) patterns using logical induction and deduction.

To improve their ability to reason mathematically, and to articulate a logical argument.

To cultivate an appreciation of the beauty, power, and mystery of mathematics in every participant.

Textbooks/ Course Readings

Required text: E.B. Burger and M. Starbird. *The Heart of Mathematics. An invitation to effective thinking.* 4th ed. John Wiley and Sons, Hoboken, 2013. Purchase or rent used, new or electronic copies at the UofT bookstore or by searching 'Starbird' at the publishers website wiley.com/en-ca Make sure to get the 4th (rather than the 3rd or earlier) edition.

Excerpts from 'In Process' by Peter Taylor (Queen's University). To be posted on the Quercus course website when needed with the author's permission.

How this course is organized:

This course will consist of one weekly lecture held 14h10-17h00 (Toronto time, usually with a 15 break somewhere in the middle) on Thursdays. Due to the Covid-19 pandemic, all components of the course will be delivered online synchronously on Zoom. Lectures will be recorded with links posted on the course website at Quercus following each class. You may experiment to see whether you find it more helpful to read the assigned sections of Burger and Starbird before or after we cover the material in class (or both). Working the assigned problems is probably more crucial to your learning than reading. Students are permitted to discuss the problems with each other, provided each student writes up their own solution by themselves. To get the most of the problems it is ESSENTIAL that you attempt them ALONE before consulting with peers.

Homework will be turned in on Crowdmark. Links will be added to the course website on Quercus, and to the website I maintain at <http://www.math.toronto.edu/mccann/199/>

Technical Requirements

In order to participate in this course, students will be required to have:

- Reliable internet access. It is recommended that students have a high speed broadband connection (LAN, Cable, or DSL) with a minimum download speed of 5 Mbps.
- A computer satisfying the minimum technical requirements (<https://www.viceprovoststudents.utoronto.ca/covid-19/tech-requirements-online-learning/>)
- A microphone.

Other recommended items include headphones, a tablet or printer, and especially a webcam.

If you are facing financial hardship, you are encouraged to contact your college or divisional registrar (<https://future.utoronto.ca/current-students/registrars/>) to apply for an emergency bursary.

III. Evaluation/ Grading Scheme

Mark Breakdown

Attendance and participation	10%
Electronic journals (Jan 29, Feb 26, Mar 12, April 3)	10%
Assignments and Quizzes	40%
Final Project	40%

Assignments

There will be an assignment each week (11 total), of which the one with lowest grade will be dropped (late submissions will be graded only at the pleasure of the professor; if graded they will be penalized 5% per day up to three days late; assignments received more than three days late receive a mark of zero unless prior permission of the instructor is obtained). Assignments are due before midnight on the evening of the Wednesday following the class in which they are assigned.

Quizzes

One or more surprise quizzes will take place in lectures during the semester. The quizzes, taken together, will have the same value as an assignment.

Final Project

Timeline: Settle on a topic and in consultation with me: Feb 12 (10%)
 Turn in a complete, full length rough draft: Mar 19 (30%)
 Revised final draft due: April 9 (60%)

Each stage of the project will be penalized by 5% per day late, up to a maximum of five days, after which the grade for that stage will be zero.

This project is intended to encourage you to begin to explore mathematics independently, and pursue some topic of interest to you within the framework of the course. Your challenge will be to address the question "What is mathematics?", and to illustrate by investigating and discussing in depth a particular mathematical topic of your choice, explaining how it supports your answer. Books listed among the accompanying references, such as "The Heart of Mathematics" or "The Princeton Companion to Mathematics" provide one set of possible sources for students in search of topics.

Summarize your findings in a double-spaced 6-8 page typed essay in logically organized and clearly written, well-constructed sentences, paragraphs and sections. Your report should include references to several articles or books that you researched while preparing it. You should have identified at least two of these sources to me by the time of our Feb 12 consultation (which can be by email but is more usefully carried out in short Zoom appointment). I will provide you with feedback on your rough draft. You may also benefit from getting feedback on both your drafts from the writing center below or linked to the course webpage before you hand them in. Schedule appointments early, as the writing centers often book up far in advance.

IV. Course Policies

Policy on Missed Term Work

As flexibility for missed or late course assignments have been built into the marking scheme, apart from the exceptions noted above late and missed assignments and/or quizzes will not be accepted for any reason.

Please note that Verification of Illness forms (also known as a "doctor's note") are temporarily not required. Students who are absent from class for any reason (e.g., COVID, cold, flu and other illness or injury, family situation) and who require consideration for missed academic work should inform the professor and report their absence through the online absence declaration. The declaration is available on ACORN under the Profile and Settings menu.

Turnitin

Turnitin may be used for detecting plagiarism in the April 9 submission of your final project. Normally, students will be required to submit written work to Turnitin.com for a review of textual similarity and detection of possible plagiarism. In doing so, students will allow their essays to be included as source documents in the Turnitin.com reference database, where they will be used solely for the purpose of detecting plagiarism. The terms that apply to the University's use of the Turnitin.com service are described on the Turnitin.com web site. If for any reason you do not wish to use Turnitin, you have the option of emailing your final project to me and providing an explanation of why you prefer not to use Turnitin.

Email Policy

Should you have a question that is not answered on the course site (please check there first!) please note that all communications with the Course Instructor or TA's must be sent from your official utoronto email

address, with the course number 199 included in the subject line. If these instructions are not followed, your email may not be responded to. Please send a reminder if the instructor does not respond to your email within 72 hours.

Online Etiquette/Participation

Students are expected to contribute to class discussions. No matter what form our connection takes I expect that everyone will treat others in a polite and respectful manner. I intend to nurture an environment that challenges your ways of thinking and allows you to reexamine your beliefs. This is difficult work. Any student that puts another person down or shows disrespect will not be allowed to participate.

Please use the raise hand feature of zoom (or put a note in the chat) and wait for the instructor to acknowledge you. However, if you have raised your hand or put a question in the chat and the instructor does not acknowledge you within a reasonable time, please don't hesitate to speak up. Since this is a seminar course, I encourage you to keep your webcams on. If you are hesitant to use your camera, you may wish to explore Zoom backgrounds as a way to disguise your surrounds but so that others can still see your face. However, if you do not want your photo to appear in course recordings, please turn off your camera before speaking.

If you cannot make a class meeting please email me before class or as soon as possible after class. It is not unexpected that some students will need to miss a class at some point during the semester. Provided I receive a timely and satisfactory explanation, your first absence will not adversely affect your participation grade in the course.

V. Institutional Policies and Support

Academic Integrity

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters (<https://governingcouncil.utoronto.ca/secretariat/policies/code-behaviour-academic-matters-july-1-2019>). If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to your Course Instructor. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources (for example, the University of Toronto website on Academic Integrity <http://academicintegrity.utoronto.ca/>).

Copyright

This course, including your participation, will be recorded on video and will be available to students in the course for viewing remotely and after each session.

Course videos and materials belong to your instructor, the University, and/or other sources depending on the specific facts of each situation and are protected by copyright. Do not download, copy, or share any course or student materials or videos without the explicit permission of the instructor.

For questions about the recording and use of videos in which you appear, please contact your instructor.

Accessibility

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a

collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs. Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach your Course Instructor and/or the Accessibility Services office as soon as possible. The sooner you let us know your needs the quicker we can assist you in achieving your learning goals in this course.

Link to Accessibility Services website: <https://studentlife.utoronto.ca/department/accessibility-services/>

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities. Services and Support

Other Academic and Personal Supports

- Writing Centre <https://writing.utoronto.ca/writing-centres/arts-and-science/>
- U of T Libraries <https://oneseach.library.utoronto.ca/>
- Feeling Distressed? <https://studentlife.utoronto.ca/task/support-when-you-feel-distressed/>
- Academic Success Centre <https://studentlife.utoronto.ca/department/academic-success/>
- College/Faculty Registrars <https://future.utoronto.ca/current-students/registrars/>
- Student Study Groups <https://sidneysmithcommons.artsci.utoronto.ca/recognized-study-groups>

TO HELP LEARNING AND TO COMBAT PANDEMIC ISOLATION, I ENCOURAGE EVERY STUDENT TO VOLUNTEER TO LEAN OR JOIN A RECOGNIZED STUDY GROUP FOR THIS COURSE USING THE ABOVE LINK!

VI. Tentative Schedule of Lectures

Week#; Section# from Burger and Starbird's "The Heart of Mathematics" 4th Ed.

NUMBERS AND COUNTING

W1 S2.1 The pigeonhole principle, estimation, and quantitative reasoning

W2 S2.3, 2.6-2.7 Primes, rationals, irrationals and real numbers

INFINITY

W3 S3.1-3.3 'The buddy system:' uncountability of the the irrationals

GEOMETRY AND TOPOLOGY

W4 S5.1-5.2 Topological equivalence; Mobius strips; classifying surfaces

W5 S4.5, 6.2 Platonic solids and the Euler characteristic proof that there are only five.

ENUMERATIVE GEOMETRY

W6 Cut Plane: "Space can be divided into two regions by a single plane, four regions by a pair of planes, and eight regions by using three planes. What is the maximum number of regions that space can be divided into by using k planes?"

OPTIMIZATION

W7 The spider and the ant: "A spider and an ant occupy a $12 \times 12 \times 24$ room. If the spider is in one corner of the room, where should the ant position himself to maximize his crawling distance from the spider? The opposite corner is an obvious guess, and is the farthest point away as the crow flies. But a spider is not a crow..."

FRACTALS

W8 S7.1-7.4 Iterated maps; Conway's Game of Life; Cantor middle thirds set; fractal dimensions, Complex numbers; complex dynamics; Julia and Mandelbrot sets

PROBABILITY AND STATISTICS

W9 S8.1-8.2 The Monte Hall problem; probability and risk

W10 S8.4-8.5 Game theory, Nash equilibria

W11 S2.2 Patterns and proofs

DECISION THEORY AND SOCIAL CHOICE

W12 S10.1-2, 10.5 Fair-allocation of scarce resources: envy-free division of cake
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