

STUDENT SEMINAR ON VARIATIONAL PROBLEMS IN PHYSICS, ECONOMICS, AND GEOMETRY

Lecture Hours: Friday 15h10-18h00 BA 6183 (including Dec 7 or 8 if necessary)
Office Hours: Monday 17h05-17h50 BA 6124

Readings: Linked at website above.

Recommended overviews (also linked):

R.J. McCann and Nestor Guillen “Five Lectures on Optimal Transportation” (2013) 145-180.
F. Santambrogio “Optimal Transport for Applied Mathematicians” Birkhauser 2015

The theme of this course will be the interplay between mathematical analysis and its application to problems arising in physics, economics, and geometry. It will be a true topics course, in the sense that its focus will be a series of problems with classical roots yet strong connections to ongoing research — optimal transportation of mass in curved landscapes, minimal and other surfaces with prescribed curvature, pattern formation, entropy, mechanism design — and their connections with classical analysis: invariance of domain and the maximum principle, minimax theorems and convex duality, calculus of variations, Hodge theory, differential equations, symmetry, regularity, geometric inequalities, continuity and compactness in infinite dimensions. Much of the relevant analysis will be developed in parallel with the applications, and it is to be hoped that the physical motivation will provide deeper insight into the mathematics.

The material will be presented by the students, based on readings assigned at the beginning of each semester. The format for the course will be as follows:

1) Readings will be assigned at the beginning of each semester; each student will be asked to present one (or more) reading per semester.

2) All students are responsible for doing the primary and alternate readings assigned in advance of each week’s presentation. They are responsible for familiarizing themselves with the issues addressed by the article(s) and their main conclusions.

3) Before each lecture, all students other than the presenter are required to submit a one paragraph written summary of the article(s) to be discussed that day, along with two questions they have formulated about the article. This summary can also be submitted by E-mail in advance.

4) The student presenting is responsible for having read and digested their article as completely as possible, at least one week prior to the presentation. It is MANDATORY that they make an appointment to meet with the instructor outside of class, no later than one week in advance of their presentation to rehearse their presentation and resolve difficulties. (Suggested times: a multihour block, starting e.g. Thursday 14h10 in BA 6124). They should come to this meeting prepared with a written outline of the structure of their presentation. Failure to arrange this meeting or inadequate preparation will be reflected in the presentation grade.

5) Some of these readings are DIFFICULT. Begin preparing for your presentation AS FAR AHEAD AS POSSIBLE. You are welcome to consult me at any time.

Grading Scheme:

50% Presentations

30% Written Summaries of Other Readings

20% Class Attendance and Active Participation

Late summaries will not be accepted (without medical justification); the grade will be zero.

The university requires students to be informed that all course marks are tentative until approved by the Department chair and Dean’s office, and recorded in the office of the faculty registrar.

Tips on presenting mathematics:

Here are some presentation tips, based on my experiences teaching similar courses in previous years.

Try not only to present the technical details correctly, but also to frame the material in an insightful or inspiring way, providing perspective on it and highlighting its interest. Far too many presentations begin with a few blackboards establishing notation, before launching into the statement and proof of Lemma 1, neither preceded by an overall statement or picture about where we are trying to get to and why, how we shall get there, and what will be the challenges and landmarks that we may expect to see along the way, nor painting in broad brush strokes the role of this lemma and how to conceive it.

A lecture is analogous on one hand to a concert performance, and on the other hand to an essay. It needs to have a beginning and an end, which recapitulate what will be said and what has been said, and several intermediate parts, each of which has its own beginning and end, which act as guideposts announcing and summarizing the interior of that part, with additional such nested structures on further levels as needed (as in a computer program). It is not enough to merely play the notes, they must be played with intelligence, sensitivity, and feeling, weaving together different themes to achieve a cohesive interpretation of the whole work of art. Not every technical detail needs to be included; the artist needs to make judgements about what to include and what to discard, and at what level of detail. Of course, this cannot be achieved unless the artist has memorized, internalized, and digested the notes to the point that they are second nature, so that he can focus on interpreting them. And this takes hard work and practice!

Similarly, when you are reading a paper, it is important to work out enough simple examples of each step to allow yourself to build up a mental image of that step which can be easily remembered, shared with the class, and related to images of the other steps to form an overall picture.

One of the problems that I have noticed in the past is that preparations for these presentations are not always started far enough in advance. Most of these papers are difficult and often require the presenter to research the literature around the reading (including references from the reading, or supplementary materials listed on the syllabus) to be able to correct omitted or erroneous steps in the original sources. Your meeting with me one week before your presentation is not generally the time to come to my office seeking clarification on a long list of points that could not be understood, any more than the dress rehearsal for a concert or play is a time for the performers to be learning their notes or their lines. You are welcome to come to my office hours for technical assistance, but ideally this should be done two or more weeks in advance of your presentation. By the week before your presentation I expect you to have complete technical control of the material, so you can present it to me as a cohesive whole, having made some judgements about what is conceptually important and what is merely technical, and so that I can help to shape the presentation by sharing context and insights I may be aware of, critiquing your representation and interpretation of the reading, and augmenting it with ideas of my own.

Of course, this will not be possible with the first few presentations in the semester, but apart from these I will count on everyone to begin researching their reading early enough that they are able to master all the technical details at least two weeks in advance of the presentation, in consultation with me as necessary, and to come to me one week in advance with a performance of the presentation ready for us to polish and shape.