# Technical Report to accompany: On the intersection problem for Steiner triple systems of different orders 

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## Introduction

This technical report is intended as a companion to On the intersection problem for Steiner triple systems of different orders by Danziger, Dukes, Griggs and Mendelsohn. For completeness we have included the paper first, followed by the detailed computational results of the following special cases:

- Cases from Lemma 16 (iv): $b-2, b-3 \in I(27,31)$.
- Cases from Lemma 16 (v): $b-2, b-3 \in I(33,37)$.
- Cases from Lemma 22 (ii): $I(13,25)=[0,22]$.
- Cases from Lemma 24 (ii): $I(15,27)=[0,27]$.
- Cases from Theorem 28: 2, 4, 5, $6 \in I(7,15), I(7,19), I(7,25)$;
$5,7,8,9,10,11 \in I(9,19), I(9,21), I(9,25), I(9,27)$
All of the results were obtained by modification of the standard hill climbing algorithm for triple systems. See section 2.72 of [4].


## Part I

## Theoretical background

## 1 Introduction

A Steiner triple system of order $v$, abbreviated $\operatorname{STS}(v)$, is a pair $(V, \mathcal{B})$ with $V$ a $v$-set of points and $\mathcal{B}$ a set of 3-subsets of $V$, called blocks or triples, such that every pair of distinct elements of $V$ occurs in exactly one triple. A necessary and sufficient condition for the existence of $\operatorname{STS}(v)$ is that $v \equiv 1$ or $3 \bmod 6$. Such orders $v$ are said to be admissible. The number of blocks of an $\operatorname{STS}(v)$ is $v(v-1) / 6$.

A partial triple system of order $v$, or $\operatorname{PTS}(v)$ is a pair $(V, \mathcal{B})$ as above, with every pair of distinct elements from $V$ occurring in at most one triple. A PTS $(u)$, say $(U, \mathcal{B})$, is said to embed in a $\operatorname{PTS}(v)$, say $\left(V, \mathcal{B}^{\prime}\right)$ if $U \subseteq V$ and $\mathcal{B} \subseteq \mathcal{B}^{\prime}$. Of particular relevance to this paper is the Pasch configuration, which is a $\operatorname{PTS}(6)$ with blocks of the form $\mathcal{B}=\{\{a, b, x\},\{a, c, y\},\{b, c, z\},\{x, y, z\}\}$. A Pasch trade swaps $x$ with $c$; this covers the same set of pairs, but with different blocks. A 1-factorization of a graph $G$ is a partition of the edges of $G$ into spanning collections of disjoint edges, called 1-factors. Note that a graph admitting a 1factorization is regular and has chromatic index equal to its degree. For further details on Pasch trades, 1-factorizations, and other basic facts regarding triple systems, the reader is referred to the relevant sections of Triple Systems by Colbourn and Rosa, [4].

For $u \leq v$, let $I(u, v)$ be the set of all $x$ such that there exists $\operatorname{STS}(u)$ and $\operatorname{STS}(v)$, say $(U, \mathcal{B})$ and $\left(V, \mathcal{B}^{\prime}\right)$ with $U \subseteq V$, and $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=x$. In [9], Lindner and Rosa essentially finish determining the sets $I(v, v)$.

Theorem 1. ([9]) Suppose $v$ is admissible and let $b=v(v-1) / 6$. Then

$$
I(v, v)= \begin{cases}\{0,1,2,3,4,6,12\} & \text { if } v=9 \\ {[0, b] \backslash\{b-1, b-2, b-3, b-5\}} & \text { otherwise }\end{cases}
$$

In this paper, we obtain some preliminary results on $I(u, v)$ when $u<v$. In what follows, $u$ and $v$ are assumed to be admissible and $b$ is used to denote $u(u-1) / 6$, the number of blocks in the smaller system.

Other standard notation used in this paper is as follows. If $A$ and $B$ are sets of integers, $A+B$ denotes the set of all sums $a+b$, where $a \in A$ and $b \in B$. The set $A-B$ is defined analogously. If $k$ is a positive integer, $k * A$ is defined as the $k$-fold sum $A+\cdots+A$. Write $[a, b]$ for the set of integers $x$ with $a \leq x \leq b$.

In $[7]$, the maximum value of $I(u, v)$ is investigated. The necessary conditions were also investigated and are summarized below.

Lemma 2. ([7]) For $u \leq v \leq 2 u+1$,

$$
I(u, v) \subseteq \begin{cases}{[0, b-(v-u)(2 u+1-v) / 6]} & \text { if } v-u \geq 4 \\ {[0, b-(u-1) / 2]} & \text { if } v-u=2 \text { and } u \neq 13 \\ {[0, b-7]} & \text { if } u=13, v=15\end{cases}
$$

In various cases, the upper bounds from Lemma 2 are shown in [7] to be achievable.

It is shown in [6] that $I(t, t) \subset I(u, v)$ for admissible $t \leq v-u-1$ and $(u-1) / 2$. In particular, for $u<v$ there is always an $\operatorname{STS}(u)$ and an $\operatorname{STS}(v)$ which are disjoint.

Lemma 3. ([6]) For $u<v, 0 \in I(u, v)$.
In the next section we give the main recursive construction. We then go on to consider the cases $v-u=2,4$ and $v=2 u-1,2 u-3$ in detail. In the nature of a recursive construction it is necessary to find many small cases, however including every case can become overwhelming. We have adopted the approach that any case needed for the recursion is included, whereas special cases have been collected together in a Part II

## 2 Main Construction

A group divisible design with block size 3, or 3-GDD, of type $T=g_{1} g_{2} \ldots g_{u}$ is a pair $(X, \mathcal{A})$ with $X$ a set of points partitioned into sets of size $g_{1}, g_{2}, \ldots, g_{u}$ called groups and $\mathcal{A}$ a set of triples from $X$ such that every pair of points in distinct groups belongs to exactly one triple. When all $g_{i}$ are equal, the 3-GDD is said to be uniform. For the type of a 3-GDD, we may use exponential notation $g^{u}$ to denote $u$ groups, each of size $g$.

Note that an $\operatorname{STS}(v)$ is a 3 -GDD of type $1^{v}$. A 3-GDD of type $1^{v-h} h^{1}$ is sometimes said to be an STS having a hole; we abbreviate this object as
$\operatorname{ISTS}(v ; h)$. When $h=1$, this is simply an $\operatorname{STS}(v)$; when $h=3$, this is an STS $(v)$ missing one block. A 3-GDD of type $n^{3}$ is equivalent to a Latin square of side $n$ and by abuse of notation is called a Latin square. A Latin square of side $n$ exists for every $n$.

For later use, we summarize some other known existence results on 3-GDDs. The comprehensive reference [3] can be consulted for details.

Lemma 4. For every $t \geq 3$, there exist $3-G D D$ s of type $6^{t}, 12^{t}, 6^{t} 8^{1}, 6^{t} 12^{1}$, and $12^{t} 6^{1}$. For $t \not \equiv 2(\bmod 3), t \geq 3$, there exists a $3-G D D$ of type $4^{t} 6^{1}$.

Write $I(3 \mathrm{GDD}(T))$ for the set of intersection numbers of two 3-GDDs of the same type $T$ on the same points. Intersections of 3 -GDDs have been investigated in [2].

Lemma 5. ([2]) If $6 \mid g$, then $I\left(3 \mathrm{GDD}\left(g^{t}\right)\right)=[0, b] \backslash\{b-1, b-2, b-3, b-5\}$, where $b=g^{2} t(t-1) / 6$.

Define $I_{h \subset h^{\prime}}(u, v)$ to be the set of all values, $x$ say, such that there exist $\operatorname{ISTS}(u ; h),(U, H, \mathcal{B})$, and $\operatorname{ISTS}\left(v ; h^{\prime}\right),\left(V, H^{\prime}, \mathcal{B}^{\prime}\right)$, where $U \subseteq V, H \subseteq H^{\prime}$, and $U \cap H^{\prime}=H$ with $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=x$.

Lemma 6. $I(u, v) \supseteq I_{h \subset h^{\prime}}(u, v)+I\left(h, h^{\prime}\right)$.
Proof. Suppose there exist $\operatorname{ISTS}(u ; h),(U, H, \mathcal{B})$, and $\operatorname{ISTS}\left(v ; h^{\prime}\right),\left(V, H^{\prime}, \mathcal{B}^{\prime}\right)$, where $U \subseteq V, H \subseteq H^{\prime}$, and $U \cap H^{\prime}=H$ with $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=x$. Suppose there exist $\operatorname{STS}(h),(H, \mathcal{A})$, and $\operatorname{STS}\left(h^{\prime}\right),\left(H^{\prime}, \mathcal{A}^{\prime}\right)$, with $\left|\mathcal{A} \cap \mathcal{A}^{\prime}\right|=y$. Then $(U, \mathcal{B} \cup \mathcal{A})$ is an $\operatorname{STS}(u)$ and $\left(V, \mathcal{B}^{\prime} \cup \mathcal{A}^{\prime}\right)$ is an $\operatorname{STS}(v)$, and these have exactly $x+y$ triples in common.

Note that the proof of Lemma 6 fails if the condition $U \cap H^{\prime}=H$ is dropped from the definition of $I_{h \subset h^{\prime}}(u, v)$, since then there may be blocks in $\mathcal{B} \cap \mathcal{A}^{\prime}$.

Theorem 7. Suppose there is a 3-GDD of type $T=g_{1} \cdots g_{t}$ and let $s<g_{i}$ for every $i$ be such that $s, g_{i}+s$, and $g_{i}+2 s+1$ are admissible for $i=1, \ldots, t$. Then

$$
\begin{aligned}
& I\left(\Sigma_{i} g_{i}+s, \Sigma_{i} g_{i}+2 s+1\right) \\
& \quad \supseteq I(3 \operatorname{GDD}(T))+\sum_{i} I_{s \subset 2 s+1}\left(g_{i}+s, g_{i}+2 s+1\right)+I(s, 2 s+1) .
\end{aligned}
$$

Proof. Since $s$, and hence $2 s+1$, are admissible and in different congruence classes $\bmod 6,6 \mid g_{i}$ for all $i$. Take two 3 -GDDs intersecting in $\alpha$ blocks on the same set of points $X$ with partition $X=\cup X_{i}$ with $\left|X_{i}\right|=g_{i}$ for each $i$. Construct $\operatorname{ISTS}\left(g_{i}+s ; s\right)$ on $X_{i} \cup Y$ and an $\operatorname{ISTS}\left(g_{i}+2 s+1 ; 2 s+1\right)$ on $X_{i} \cup Y^{\prime}$ with holes $Y \subset Y^{\prime}$, respectively. Suppose these ISTS intersect in $\beta_{i}$ blocks. Finally place subsystems on $Y$ and $Y^{\prime}$ which intersect in $\gamma$ blocks. The result is an $\operatorname{STS}\left(\Sigma_{i} g_{i}+s\right)$ intersecting an $\operatorname{STS}\left(\Sigma_{i} g_{i}+2 s+1\right)$ in $\alpha+\sum_{i} \beta_{i}+\gamma$ blocks.

An obvious drawback of the main construction is the current lack of knowledge about $I_{h \subset h^{\prime}}(u, v)$ and $I(3 \mathrm{GDD}(T))$ for non-uniform 3-GDD types $T$. The following application of Wilson's fundamental construction allows us to get a partial result towards the latter question.

Theorem 8. Suppose $T=g_{1} \ldots g_{t}$ is the type of a 3-GDD with b blocks and for $m \geq 1$, let $m T$ denote the type $\left(m g_{1}\right) \ldots\left(m g_{t}\right)$. Then

$$
I(3 \mathrm{GDD}(m T)) \supseteq b * I\left(3 \mathrm{GDD}\left(m^{3}\right)\right)
$$

Proof. Consider a 3-GDD of type $T$ with blocks $B_{1}, \ldots, B_{b}$. Give every point weight $m$. Replace $B_{i}=\{x, y, z\}$ with the set of blocks of a 3-GDD of type $m^{3}$ whose three groups correspond to the weightings of points $x, y, z$. For each $i$, this can be done with any $\beta_{i} \in I\left(3 \mathrm{GDD}\left(m^{3}\right)\right)$ common blocks. Thus we obtain two 3-GDDs of type $m T$ intersecting in $\sum_{i=1}^{b} \beta_{i}$ blocks.

## Corollary 9.

(i) $I\left(3 \operatorname{GDD}\left(12^{t} 18^{1}\right)\right) \supseteq\{0,3,6, \ldots, 24 t(t+2)-6,24 t(t+2)\}$ for $t \not \equiv 2(\bmod 3)$ and
(ii) $I\left(3 \mathrm{GDD}\left(18^{t} 24^{1}\right)\right) \supseteq\{0,3,6, \ldots, 18 t(3 t+5)-6,18 t(3 t+5)\}$ for all $t \geq 3$.

Proof. Note that $I\left(3 \operatorname{GDD}\left(3^{3}\right)\right)=\{0,3,9\}$ and $b *\{0,3,9\}=\{0,3,6, \ldots, 9 b-$ $6,9 b\}$. For (i) apply Theorem 8 with $m=3$ and $T=4^{t} 6^{1}$. The number of blocks of a 3-GDD of type $T$ is $8 t(t+2) / 3$. So $I\left(12^{t} 18^{1}\right) \supseteq 8 t(t+2) / 3 *\{0,3,9\}$. For (ii) let $m=3$ and $T=6^{t} 8^{1}$. The number of blocks of a 3-GDD of type $T$ is $2 t(3 t+5)$. So $I\left(18^{t} 12^{1}\right) \supseteq 2 t(3 t+5) *\{0,3,9\}$.

## $3 \quad v-u=2$

Lemma 10. $I(7,9)=\{0,1,2,4\}$.
Proof. By Theorem 2, we have $I(7,9) \subseteq[0,4]$. Consider the $\operatorname{STS}(9)$ on points $\{\infty, 1, \ldots, 6, A, B\}$ given below.

| $\{\infty, A, B\}$ | $\{\infty, 1,4\}$ | $\{\infty, 2,6\}$ | $\{\infty, 3,5\}$ |
| :---: | :---: | :---: | :---: |
| $\{1,2,3\}$ | $\{A, 2,5\}$ | $\{A, 3,4\}$ | $\{A, 1,6\}$ |
| $\{4,5,6\}$ | $\{B, 3,6\}$ | $\{B, 1,5\}$ | $\{B, 2,4\}$ |

Any $\operatorname{STS}(7)$ on $\{\infty, 1, \ldots, 6\}$ which intersects this in three blocks must do so in three of the blocks $\{1,2,3\},\{4,5,6\},\{\infty, 1,4\},\{\infty, 2,6\},\{\infty, 3,5\}$. Because the first two are disjoint, at least two of the blocks must contain $\infty$. Then the STS(7) must contain the last three of these blocks. But the system can then not be completed without either of the blocks $\{1,2,3\}$ or $\{4,5,6\}$.

This proves $I(7,9) \subseteq\{0,1,2,4\}$. Examples of $\operatorname{STS}(7)$ meeting the above $\operatorname{STS}(9)$ in $0,1,2$, and 4 blocks are given below.

$$
\begin{aligned}
& \{\infty, 1,2\},\{\infty, 3,4\},\{\infty, 5,6\},\{1,4,6\},\{1,3,5\},\{2,3,6\},\{2,4,5\} \\
& \{\infty, 1,5\},\{\infty, 2,4\},\{\infty, 3,6\},\{1,2,3\},\{1,4,6\},\{2,5,6\},\{3,4,5\} \\
& \{\infty, 1,4\},\{\infty, 2,5\},\{\infty, 3,6\},\{1,2,3\},\{1,5,6\},\{2,4,6\},\{3,4,5\} \\
& \{\infty, 1,4\},\{\infty, 2,6\},\{\infty, 3,5\},\{1,2,3\},\{1,5,6\},\{2,4,5\},\{3,4,6\}
\end{aligned}
$$

By using computer search, we have determined $I(13,15)$.
Lemma 11. $I(13,15)=[0,19]$.
Proof. Consider the $\operatorname{STS}(13)(U, \mathcal{B})$ with blocks

```
{0,1,2},{0,3,4},{0,5,6},{0,7,8},{0,9,10},{0,11,12},{1,3,5},{1,4,7},{1,6,8},
{1,9,11},{1,10,12},{2,3,9},{2,4,5},{2,6,10},{2,7,12},{2,8,11},{3,6,11},{3,7,10},
{3,8,12},{4,6,12},{4,8,9},{4,10,11},{5,7,11},{5,8,10},{5,9,12},{6,7,9}
```

Below are blocks $\mathcal{B}^{\prime}$ for an $\operatorname{STS}(15)$ on $U \cup\{A, B\}$ having 19 blocks in common with $\mathcal{B}$. We have separated these blocks according to their intersection with $\mathcal{B}$.
$\{0,1,2\},\{0,3,4\},\{0,7,8\},\{0,11,12\},\{1,3,5\},\{1,4,7\},\{1,6,8\},\{1,9,11\},\{1,10,12\}$, $\{2,4,5\},\{2,6,10\},\{2,8,11\},\{3,6,11\},\{3,7,10\},\{3,8,12\},\{4,6,12\},\{5,7,11\},\{5,8,10\}$, $\{6,7,9\}$
$\{0,5,9\},\{2,9,12\},\{4,9,10\},\{3,9, A\},\{1, A, B\},\{5,12, A\},\{0,6, A\},\{2,7, A\},\{0,10, B\}$, $\{2,3, B\},\{10,11, A\},\{7,12, B\},\{5,6, B\},\{8,9, B\},\{4,8, A\},\{4,11, B\}$

We now give three Pasch configurations in $\mathcal{B}^{\prime}$. These cover disjoint pairs and have 1,2 and 4 blocks in common with $\mathcal{B} \cap \mathcal{B}^{\prime}$, repsectively.

```
{0,5,9},{0,10,B},{5,8,10},{8,9,B}
{4,9,10},{4,6,12},{2,9,12},{2,6,10}
{0,1,2},{0,3,4},{1,3,5},{2,4,5}
```

Applying Pasch trades to zero or more of these, we can decrease the intersection with $\mathcal{B}$ by any of $0,1,2, \ldots, 7$. Thus $[12,19] \subseteq I(13,15)$.

To show $11=19-8 \in I(13,15)$, we take two (edge-disjoint) Pasch configurations with 8 total blocks in $\mathcal{B} \cap \mathcal{B}^{\prime}$ and trade each.

```
{0,3,4},{0,11,12},{3,6,11},{4,6,12}
{1,3,5},{1,10,12},{3,8,12},{5,8,10}
```

Although Pasch trades are more difficult to construct when $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|$ is smaller, we can demonstrate $[0,10] \subseteq I(13,15)$ with the same $\operatorname{STS}(13)$ above and by direct construction of various STS(15).

```
{6,10,A},{0,5,7},{7,A,B},{1,5,B},{0,6,B},{3,5,12},{0,11,A},{1,8,A},{1,4,10},
{0,3,10},{8,10,11},{2,3,A},{4,12,A},{1,3,11},{2,8,B},{4,5,11},{0,1,9},{3,4,B},
{8,9,12},{2,5,10},{5,6,8},{5,9,A},{0,2,12},{4,6,7},{3,7,8},{1,6,12},{0,4,8},
{7,9,10},{1,2,7},{10,12,B},{7,11,12},{2,4,9},{9,11,B},{3,6,9},{2,6,11}
{5,7,11}
{3,4,11},{1,6,7},{10,11,B},{0,3,6},{6,12,B},{3,7,B},{9,A,B},{2,4,B},{0,9,12},
{2,6,A},{7,8,10},{8,11,12},{1,5,B},{5,8,9},{3,9,10},{4,5,12},{1,3,12},{7,12,A},
{0,1,10},{3,8,A},{5,6,10},{1,11,A},{1,2,8},{0,4,7},{2,7,9},{0,8,B},{6,9,11},
{0,2,11},{1,4,9},{2,3,5},{4,6,8},{2,10,12},{4,10,A},{0,5,A}
```

$\{5,9,12\},\{1,4,7\}$
$\{2,8, B\},\{4,8,12\},\{3,4,5\},\{2,3, A\},\{6,10, A\},\{3,9,11\},\{1,3,10\},\{0,2,6\},\{2,5,10\}$, $\{4, A, B\},\{10,11,12\},\{0,8,11\},\{1,2,12\},\{6,12, B\},\{7,12, A\},\{1,11, B\},\{5,11, A\}$, $\{8,9, A\},\{0,4,10\},\{3,6,8\},\{5,6,7\},\{0,5, B\},\{4,6,11\},\{2,7,11\},\{9,10, B\},\{1,6,9\}$, $\{0,7,9\},\{2,4,9\},\{0,1, A\},\{1,5,8\},\{7,8,10\},\{0,3,12\},\{3,7, B\}$
$\{0,9,10\},\{1,10,12\},\{4,6,12\}$
$\{0,1,4\},\{1,6, A\},\{0,6,11\},\{4,8,10\},\{0,2,8\},\{2,3,12\},\{0,7, A\},\{4,5,11\},\{7,11, B\}$, $\{5,8, B\},\{0,3, B\},\{3,10,11\},\{3,6,7\},\{1,3,8\},\{8,11, A\},\{0,5,12\},\{6,10, B\},\{3,4, A\}$, $\{9,11,12\},\{1,2,11\},\{7,8,12\},\{1,5,7\},\{12, A, B\},\{4,7,9\},\{2,5,6\},\{5,10, A\},\{6,8,9\}$, $\{3,5,9\},\{2,7,10\},\{2,9, A\},\{1,9, B\},\{2,4, B\}$
$\{4,6,12\},\{1,4,7\},\{5,7,11\},\{3,6,11\}$
$\{0,1,5\},\{1,3,12\},\{2,6,9\},\{1,8, B\},\{0,9,12\},\{3,7,9\},\{4,8, A\},\{2,12, A\},\{5,8,12\}$, $\{2,5,10\},\{6,7,8\},\{0,6, B\},\{0,2,7\},\{5,6, A\},\{0,4,11\},\{9,10, B\},\{4,5,9\},\{7,10, A\}$, $\{3,5, B\},\{2,3,8\},\{0,3, A\},\{3,4,10\},\{7,12, B\},\{2,4, B\},\{1,2,11\},\{8,9,11\},\{1,9, A\}$, $\{10,11,12\},\{11, A, B\},\{0,8,10\},\{1,6,10\}$
$\{1,4,7\},\{0,3,4\},\{2,6,10\},\{0,7,8\},\{1,6,8\}$
$\{2,5,9\},\{0,5,10\},\{0,6,12\},\{7,10,12\},\{1,12, A\},\{4,5,12\},\{0,2, A\},\{5,6, B\},\{4,6,9\}$, $\{5,7, A\},\{1,2,3\},\{3, A, B\},\{0,1, B\},\{2,7,11\},\{3,9,12\},\{1,9,10\},\{2,12, B\},\{2,4,8\}$, $\{8,11,12\},\{4,11, B\},\{3,10,11\},\{1,5,11\},\{7,9, B\},\{8,10, B\},\{8,9, A\},\{3,5,8\},\{3,6,7\}$, $\{4,10, A\},\{0,9,11\},\{6,11, A\}$
$\{3,6,11\},\{0,3,4\},\{4,8,9\},\{1,10,12\},\{0,9,10\},\{5,7,11\}$
$\{3,7,8\},\{1,2,3\},\{0,6,7\},\{5,6,10\},\{4,12, B\},\{2,8, A\},\{4,7,10\},\{1,4,11\},\{2,5,9\}$,
$\{1,7,9\},\{3,9, B\},\{1,6, B\},\{0,8,11\},\{2,7, B\},\{9,11,12\},\{2,4,6\},\{11, A, B\},\{2,10,11\}$, $\{6,8,12\},\{6,9, A\},\{3,10, A\},\{7,12, A\},\{3,5,12\},\{1,5,8\},\{0,1, A\},\{4,5, A\},\{0,2,12\}$, $\{8,10, B\},\{0,5, B\}$
$\{2,7,12\},\{1,10,12\},\{0,5,6\},\{1,6,8\},\{3,8,12\},\{2,4,5\},\{3,7,10\}$
$\{3,5,9\},\{0,8,9\},\{1,3,4\},\{2,3,6\},\{7,11, B\},\{0,1,11\},\{1,2, A\},\{4,9,10\},\{9,12, B\}$, $\{0,7, A\},\{6,11,12\},\{4, A, B\},\{6,9, A\},\{6,10, B\},\{0,3, B\},\{2,9,11\},\{5,7,8\},\{0,4,12\}$, $\{1,7,9\},\{5,10,11\},\{5,12, A\},\{1,5, B\},\{4,6,7\},\{4,8,11\},\{3,11, A\},\{0,2,10\},\{2,8, B\}$, $\{8,10, A\}$
$\{0,1,2\},\{0,3,4\},\{0,5,6\},\{3,8,12\},\{2,4,5\},\{1,6,8\},\{3,7,10\},\{2,3,9\}$
$\{1,7,9\},\{3,6, A\},\{1,10, A\},\{4,9, B\},\{0,8,9\},\{6,7, B\},\{8,10,11\},\{9,12, A\},\{0,7,11\}$, $\{1,3, B\},\{3,5,11\},\{2,10, B\},\{2,11, A\},\{2,6,12\},\{5,9,10\},\{11,12, B\},\{4,6,10\}$, $\{0,10,12\},\{6,9,11\},\{5,8, B\},\{2,7,8\},\{1,4,11\},\{1,5,12\},\{4,7,12\},\{0, A, B\},\{5,7, A\}$, $\{4,8, A\}$
$\{4,10,11\},\{5,7,11\},\{4,6,12\},\{0,7,8\},\{2,3,9\},\{3,8,12\},\{3,6,11\},\{4,8,9\},\{1,6,8\}$ $\{5,12, A\},\{1,12, B\},\{1,7,9\},\{3,7, A\},\{0,3, B\},\{9,11,12\},\{6,7, B\},\{2,11, B\},\{2,4,7\}$, $\{0,1,11\},\{2,8,10\},\{8,11, A\},\{3,5,10\},\{1,3,4\},\{0,6,10\},\{1,2,5\},\{7,10,12\},\{0,2,12\}$, $\{2,6, A\},\{5,8, B\},\{5,6,9\},\{9,10, B\},\{1,10, A\},\{0,9, A\},\{0,4,5\},\{4, A, B\}$
$\{4,6,12\},\{4,8,9\},\{0,5,6\},\{4,10,11\},\{3,7,10\},\{2,6,10\},\{5,9,12\},\{1,4,7\},\{1,6,8\}$, $\{1,9,11\}$
$\{0,3,9\},\{7,8, B\},\{3,8,11\},\{2,7,9\},\{1,2, A\},\{6,7,11\},\{2,11,12\},\{2,3, B\},\{1,5,10\}$, $\{3,4,5\},\{6,9, B\},\{9,10, A\}\{2,5,8\},\{3,6, A\},\{4, A, B\},\{0,2,4\},\{5,7, A\},\{10,12, B\}$, $\{0,8,10\},\{0,1, B\},\{1,3,12\},\{5,11, B\},\{0,7,12\},\{8,12, A\}\{0,11, A\}$

We now outline a general construction for values near the maximum possible intersection $b-3 t$ of $\operatorname{STS}(u)$ and $\operatorname{STS}(u+2)$, although only the two cases in Example 12 will actually be required. Let $\alpha(t)$ denote the PTS having $6 t$ points $X_{t}=\left\{a, x_{0}, y_{0}\right\} \cup\left\{x_{i}, y_{i}, z_{i}: i=1, \ldots, 2 t-1\right\}$ and $4 t-1$ blocks

$$
\mathcal{B}_{t}=\left\{\left\{a, x_{0}, y_{0}\right\}\right\} \cup \bigcup_{i=1}^{2 t-1}\left\{\left\{x_{i-1}, x_{i}, y_{i}\right\},\left\{y_{i-1}, y_{i}, z_{i}\right\}\right\}
$$



Figure 1: the PTS $\alpha(1)$ and $\alpha(2)$

The figure shows diagrams for $\alpha(1)$ and $\alpha(2)$. We note that the underlying graph of $\alpha(t)$ can be decomposed into an "outside" cycle of length $6 t$ and $2 t-1$ "inside" triples of the form $\left\{x_{i-1}, y_{i-1}, y_{i}\right\}$. Thus suppose an $\operatorname{STS}(6 t+1)$, say $(U, \mathcal{B})$, contains copies of PTS $\alpha\left(t_{1}\right), \ldots, \alpha\left(t_{s}\right)$ on disjoint sets of points, where $t=t_{1}+\cdots+t_{s}$. Let $\infty$ be the unique point not among the $\alpha\left(t_{i}\right)$. Form an $\operatorname{STS}(6 t+3)\left(V, \mathcal{B}^{\prime}\right)$ as follows: remove the blocks of the $\alpha_{i}$ (a total of $4 t-s$ ), add the inside triples, join new points $A, B$ to 1 -factors of the disjoint union of outside cycles, and include $\{\infty, A, B\}$. We have $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=b-4 t+s$. Note that certain variants on $\alpha(t)$ could be used in a similar manner.

In [7], STS containing $t$ disjoint copies of $\alpha(1)$ were used to obtain the maximum value $b-3 t$ for $I(6 t+1,6 t+3)$ when $t \neq 2$, and an STS(13) containing $\alpha(2)$ was used to obtain the maximum possible $b-3 t-1$ for $I(13,15)$.

Example 12. Using a computer to complete PTS by hill-climbing, we note that there exists an $\operatorname{STS}(19)$ containing disjoint $\alpha(1)$ and $\alpha(2)$, and an $\operatorname{STS}(25)$ containing disjoint $\alpha(1), \alpha(1)$ and $\alpha(2)$. Thus, $b-3 t-1$ belongs to both $I(19,21)$ and $I(25,27)$.

- $\operatorname{STS}(19)$ containing $\alpha(1), \alpha(2)$ :
$\{0,1,2\},\{1,3,4\},\{2,4,5\},\{4,6,7\},\{5,7,8\},\{7,9,10\},\{8,10,11\},\{12,13,14\}$, $\{14,15,16\},\{12,16,17\}$
$\{0,10,15\},\{0,5, \infty\},\{3,11,17\},\{7,15, \infty\},\{2,3,16\},\{2,8,15\},\{2,6,14\},\{5,14,17\}$, $\{7,11,14\},\{2,11, \infty\},\{0,3,12\},\{1,8,17\},\{6,9,13\},\{2,10,13\},\{8,13,16\},\{4,9,14\}$, $\{3,7,13\},\{10,16, \infty\},\{3,14, \infty\},\{4,15,17\},\{0,7,17\},\{0,4,13\},\{1,6,15\},\{1,5,13\}$, $\{3,6,8\},\{5,6,16\},\{0,6,11\},\{5,12,15\},\{1,9, \infty\},\{0,8,14\},\{2,7,12\},\{0,9,16\}$, $\{3,5,10\},\{4,8, \infty\},\{1,11,12\},\{6,10,17\},\{4,10,12\},\{5,9,11\},\{11,13,15\}$, $\{2,9,17\},\{4,11,16\},\{13,17, \infty\},\{6,12, \infty\},\{1,7,16\},\{1,10,14\},\{8,9,12\}$, $\{3,9,15\}$
- $\operatorname{STS}(25)$ containing $\alpha(1), \alpha(1), \alpha(2)$ :
$\{0,1,2\},\{1,3,4\},\{2,4,5\},\{4,6,7\},\{5,7,8\},\{7,9,10\},\{8,10,11\},\{12,13,14\}$, $\{14,15,16\},\{12,16,17\},\{18,19,20\},\{20,21,22\},\{18,22,23\}$
$\{13,17,23\},\{3,12,23\},\{0,11, \infty\},\{6,8,19\},\{9,12,19\},\{2,11,15\},\{0,8,17\}$, $\{7,11,14\},\{1,6,9\},\{7,17,20\},\{6,12,21\},\{3,7,21\},\{0,10,12\},\{7,19,22\}$, $\{6,13,22\},\{3,6,10\},\{8,14,18\},\{1,5,17\},\{2,7,23\},\{11,13,18\},\{6,11,17\}$, $\{3,13,15\},\{6,15,23\},\{3,16,18\},\{5,6,18\},\{1,19,23\},\{5,20, \infty\},\{0,16,23\}$, $\{2,19, \infty\},\{2,3,20\},\{8,23, \infty\},\{13,16,21\},\{9,13, \infty\},\{11,19,21\},\{2,12,22\}$, $\{4,10, \infty\},\{4,12,18\},\{2,8,13\},\{9,14,20\}\{1,18, \infty\},\{10,17,22\},\{0,6,20\}$, $\{5,9,22\},\{14,21,23\},\{3,9,11\},\{4,13,20\},\{10,18,21\},\{7,12, \infty\},\{0,3,19\}$, $\{8,12,20\},\{3,8,22\},\{0,4,14\},\{2,6,16\},\{2,10,14\},\{0,9,18\},\{0,15,22\},\{0,7,13\}$, $\{7,15,18\},\{4,11,22\},\{5,12,15\},\{5,13,19\},\{1,11,12\},\{5,11,16\},\{4,17,21\}$, $\{10,15,19\},\{16,22, \infty\},\{1,8,21\},\{1,14,22\},\{15,21, \infty\},\{8,9,16\},\{14,17,19\}$, $\{2,17,18\},\{9,15,17\},\{1,15,20\},\{3,5,14\},\{10,16,20\},\{4,8,15\},\{4,16,19\}$, $\{11,20,23\},\{6,14, \infty\},\{4,9,23\},\{3,17, \infty\},\{1,10,13\},\{1,7,16\},\{5,10,23\}$, $\{2,9,21\},\{0,5,21\}$

It is now possible to finish deciding $I(u, v)$ for $v-u=2$.

## Theorem 13.

$$
I(6 t+1,6 t+3)= \begin{cases}{[0, b-3 t] \backslash\{3\}} & \text { for } t=1 \\ {[0, b-3 t] \backslash\{20\}} & \text { for } t=2 \\ {[0, b-3 t]} & \text { for } t \geq 3\end{cases}
$$

Proof. The first two cases are handled in Lemmas 10 and 11. Let $t \geq 3$ and take $s=1$ and $g_{i}=6$ for $i=1, \ldots, t$. By Theorem 7, we have

$$
\begin{aligned}
& I(6 t+1,6 t+3) \supseteq I\left(3 \mathrm{GDD}\left(6^{t}\right)\right)+\sum_{i=1}^{t} I_{1 \subset 3}(7,9)+I(1,3) \\
& \quad=([0,6 t(t-1)] \backslash(\{6 t(t-1)\}-\{1,2,3,5\}))+t *\{0,1,2,4\} \\
& \quad=[0, b-3 t] \backslash\{b-3 t-1\}
\end{aligned}
$$

It remains to show that $b-3 t-1 \in I(6 t+1,6 t+3)$ for $t \geq 3$. Example 12 gives the result for $t=3$ and $t=4$. For $t \geq 5$, use Theorem 7 with 3-GDD of type $6^{t-2} 12^{1}$, which has $\frac{1}{3}\left(6^{2}\binom{t-2}{2}+6 \cdot 12(t-2)\right)=6 t^{2}-6 t-12$ blocks. Fill groups with $t-2$ pairs of $\operatorname{STS}(7)$ and $\operatorname{STS}(9)$ intersecting in 4 blocks, and one $\operatorname{STS}(13)$ and STS(15) intersecting in 19 blocks. Thus, $6 t^{2}-6 t-12+4(t-2)+19=$ $6 t^{2}-2 t-1 \in I(6 t+1,6 t+3)$.

## $4 \quad v-u=4$

The following results aid in determining $I_{3 \subset 7}(u, u+4)$.
Let $X$ be an $n$-set and consider a partition $H_{1}, \ldots, H_{k}$ of $X$ with $\left|H_{i}\right|=h_{i}$ for $i=1, \ldots, k$. Write $x \sim y$ if $x, y \in H_{i}$ for some $i$. Consider an $n \times n$ array $A$ with rows, columns, and entries indexed by $X$ such that (a) $A(x, y)$ is blank if and only if $x \sim y$ for some $i$, and (b) the entries $A(a, y)$ with $a \nsim y$ exhaust
$\{z: a \nsim z\}$. The array $A$ is called an incomplete Latin square with hole sizes $h_{1}, \ldots, h_{k}$, or ILS $\left(h_{1}, \ldots, h_{k}\right)$. As with GDDs, we may write the hole sizes of an ILS using exponential notation. We note that an $\operatorname{ILS}\left(h_{1}, \ldots, h_{k}\right)$ can be "filled" with Latin squares of sides $h_{1}, \ldots, h_{k}$ to form a Latin square of side $n$. For $t \geq 3$, it is not hard to construct an $\operatorname{ILS}\left(2^{t}\right)$ (and an $\left.\operatorname{ILS}\left(2^{t} 1^{1}\right)\right)$ by doubling a Latin square of side $t$ (and then prolonging along an off-diagonal transversal). An explicit construction of $\operatorname{ILS}\left(2^{4} 1^{1}\right)$ is given in [7].

The set consisting of the number of common (non-blank) entries to a pair of $\operatorname{ILS}\left(h_{1}, \ldots, h_{k}\right)$ on the same symbols is denoted $I\left(\operatorname{ILS}\left(h_{1}, \ldots, h_{k}\right)\right)$.
Lemma 14. Let $t \geq 3$ and suppose $2 t+1 \equiv 1,3(\bmod 6)$. Then

$$
I_{3 \subset 7}(6 t+3,6 t+7) \supseteq I\left(\operatorname{ILS}\left(2^{t} 1^{1}\right)\right)+3 * I(2 t+1,2 t+1)
$$

Proof. Take two $\operatorname{ILS}\left(2^{t} 1^{1}\right)$, say $A_{1}, A_{2}$ having point partition $X=\left\{1,1^{\prime}\right\} \cup \ldots$ $\cup\left\{t, t^{\prime}\right\} \cup\{\infty\}$ and with $\alpha$ common entries. Fill the $2 \times 2$ holes of $A_{1}$ to obtain the square $A_{1}^{*}$ of side $2 t+1$ missing the $(\infty, \infty)$ entry. Let $U=\left\{x_{r}, x_{c}, x_{e}: x \in X\right\}$ and suppose $\mathcal{B}_{r}, \mathcal{B}_{c}, \mathcal{B}_{e}$ and $\mathcal{B}_{r}^{\prime}, \mathcal{B}_{c}^{\prime}, \mathcal{B}_{e}^{\prime}$ are block sets of $\operatorname{STS}(2 t+1)$ on points $X$ with subscripts $r, c, e$, respectively, with $\left|B_{r} \cap B_{r}^{\prime}\right|=\beta_{r},\left|B_{c} \cap B_{c}^{\prime}\right|=\beta_{c}$, and $\left|B_{e} \cap B_{e}^{\prime}\right|=\beta_{e}$. Now form an $\operatorname{STS}(6 t+3)$ on the points $U$ having as blocks $\left\{\infty_{r}, \infty_{c}, \infty_{e}\right\}$ and

$$
\mathcal{B}=\left\{\left\{x_{r}, y_{c}, z_{e}\right\}: A_{1}^{*}(x, y)=z\right\} \cup \mathcal{B}_{r} \cup \mathcal{B}_{c} \cup \mathcal{B}_{e} .
$$

We also have an $\operatorname{STS}(6 t+7)$ on the points $U \cup\{A, B, C, D\}$ containing a sub$\operatorname{STS}(7)$ on $\left\{\infty_{r}, \infty_{c}, \infty_{e}, A, B, C, D\right\}$ and with other blocks

$$
\mathcal{B}^{\prime}=\left\{\left\{x_{r}, y_{c}, z_{e}\right\}: A_{2}(x, y)=z\right\} \cup \mathcal{B}_{r}^{\prime} \cup \mathcal{B}_{c}^{\prime} \cup \mathcal{B}_{e}^{\prime} \cup \mathcal{D},
$$

where $\mathcal{D}$ are blocks arising from joining four disjoint 1-factors of $K_{2,2,2}$ on $\left\{i_{r}, i_{r}^{\prime}\right\} \cup\left\{i_{c}, i_{c}^{\prime}\right\} \cup\left\{i_{e}, i_{e}^{\prime}\right\}$ to distinct points $A, B, C, D$ for $i=1, \ldots, t$. We have $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=\alpha+\beta_{r}+\beta_{c}+\beta_{e} \in I_{3 \subset 7}(6 t+3,6 t+7)$.

Corollary 15.
(i) $[0,57] \backslash\{48,52,54,55,56\} \subseteq I_{3 \subset 7}(21,25)$,
(ii) $[0,100] \backslash\{93,95,97,98,99\} \subseteq I_{3 \subset 7}(27,31)$, and
(iii) $[0,301] \backslash\{296,298,299,300\} \subseteq I_{3 \subset 7}(45,49)$

Proof. (i) By swapping up to 3 pairs of rows incident with the same hole in an $\operatorname{ILS}\left(2^{3} 1^{1}\right)$, we have $6,16,26,36 \in I\left(\operatorname{ILS}\left(2^{3} 1^{1}\right)\right)$. Swapping all pairs of corresponding rows and columns, we have disjoint $\operatorname{ILS}\left(2^{3} 1^{1}\right)$. Now use Lemma 14 to get

$$
I_{3 \subset 7}(21,25) \supseteq\{0,6,16,26,36\}+3 * I(7)=[0,57] \backslash\{48,52,54,55,56\}
$$

(ii) As above, we have $0,8,22,36,50,64 \in I\left(\operatorname{ILS}\left(2^{4} 1^{1}\right)\right)$. By swapping a single $2 \times 2$ subsquare, we also have $\operatorname{ILS}\left(2^{4} 1^{1}\right)$ agreeing in all but four entries. So by Lemma 14, we have

$$
I_{3 \subset 7}(27,31) \supseteq\{0,8,22,36,50,60,64\}+3 * I(9)=[0,100] \backslash\{93,95,97,98,99\} .
$$

(iii) The proof is similar to part (i) but with an $\operatorname{ILS}\left(2^{7} 1^{1}\right)$ and we leave the details to the reader.

## Lemma 16.

(i) $I(9,13)=[0,8]$,
(ii) $I_{3 \subset 7}(15,19)=[0,26]$,
(iii) $I_{3 \subset 7}(21,25)=[0,57]$,
(iv) $I(27,31)=[0, b-16]$,
(v) $I(33,37)=[0, b-20]$, and
(vi) $I(45,49)=[0, b-28]$.

Proof. Note first that, apart from cases (ii) and (iii), each claimed set is as large as possible, by Lemma 2. We subtract one from the maximum intersection for parts (ii) and (iii) in order to accommodate the hole.
(i) Consider the $\operatorname{STS}(9)$ with blocks
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{0,4,8\},\{1,5,6\},\{2,3,7\},\{0,5,7\}$, $\{1,3,8\},\{2,4,6\}$.

The following $\operatorname{STS}(13)$ on $\{0, \ldots, 8\} \cup\{A, B, C, D\}$ have intersections $0, \ldots, 8$ with the above system.

```
\(\{1,4, A\},\{4,6, B\},\{3,4,7\},\{2,5, D\},\{0, A, C\},\{2,3, A\},\{8, A, B\},\{0,1,8\},\{4,8, D\}\),
\(\{1,2,7\},\{0,3, D\},\{2,6,8\},\{0,4,5\},\{1,3,6\},\{2,4, C\},\{5,7,8\},\{6, C, D\},\{0,2, B\}\),
\(\{1,5, C\},\{7, B, C\},\{0,6,7\},\{5,6, A\},\{3,8, C\},\{7, A, D\},\{3,5, B\},\{1, B, D\}\)
\(\{2,4,6\}\)
\(\{2,3, D\},\{1,2, B\},\{4,7, B\},\{2,8, C\},\{3, A, C\},\{2,5, A\},\{1,3,6\},\{0,1,5\},\{1,4,8\}\),
\(\{6,7, C\},\{0,8, A\},\{1,7, A\},\{4,5, C\},\{0,3,4\},\{0, B, C\},\{0,2,7\},\{1, C, D\},\{5,7, D\}\),
\(\{4, A, D\},\{8, B, D\},\{3,7,8\},\{3,5, B\},\{0,6, D\},\{5,6,8\},\{6, A, B\}\)
\(\{0,5,7\},\{2,5,8\}\)
\(\{0,2,6\},\{0, C, D\},\{5, B, C\},\{3,4,6\},\{1, B, D\},\{0,1, A\},\{2,3, C\},\{2, A, B\},\{3,7, B\}\),
\(\{5,6, D\},\{3, A, D\},\{0,3,8\},\{1,3,5\},\{1,2,7\},\{8, A, C\},\{1,4,8\},\{7,8, D\},\{4,5, A\}\),
\(\{2,4, D\},\{0,4, B\},\{6,7, A\},\{6,8, B\},\{4,7, C\},\{1,6, C\}\)
\(\{0,5,7\},\{2,4,6\},\{1,4,7\}\)
\(\{2,7,8\},\{1,5, C\},\{3,8, A\},\{7, C, D\},\{8, B, C\},\{2,5, D\},\{0,6, C\},\{3,4, C\},\{3,5, B\}\),
\(\{7, A, B\},\{6, B, D\},\{5,6, A\},\{1,2, B\},\{3,6,7\},\{1,6,8\},\{0,1, A\},\{1,3, D\},\{0,2,3\}\),
\(\{2, A, C\},\{4,5,8\},\{4, A, D\},\{0,8, D\},\{0,4, B\}\)
\(\{0,1,2\},\{1,3,8\},\{0,5,7\},\{1,4,7\}\)
\(\{1,6, A\},\{7, A, C\},\{6,8, C\},\{1,5, C\},\{7,8, B\},\{0,3,4\},\{4, C, D\},\{2,3, C\},\{3, A, D\}\),
\(\{3,5, B\},\{3,6,7\},\{5,8, D\},\{4, A, B\},\{2,5, A\},\{1, B, D\},\{2,7, D\},\{2,4,8\},\{2,6, B\}\),
\(\{0,8, A\},\{0, B, C\},\{0,6, D\},\{4,5,6\}\)
\(\{0,5,7\},\{2,3,7\},\{6,7,8\},\{0,3,6\},\{2,4,6\}\)
\(\{2, B, D\},\{3, B, C\},\{2,8, A\},\{3,5, D\},\{4, A, D\},\{0,1,4\},\{0, A, B\},\{6, C, D\},\{1,8, C\}\),
\(\{0,2, C\},\{5,6, A\},\{1,6, B\},\{1,2,5\},\{1,3, A\},\{0,8, D\},\{4,5, C\},\{7, A, C\},\{5,8, B\}\),
\(\{3,4,8\},\{1,7, D\},\{4,7, B\}\)
```

```
{2,4,6},{1,5,6},{6,7,8},{0,3,6},{1,3,8},{2,5,8}
{8,B,D},{4,8,C},{0,1,4},{4,7,B},{6,B,C},{2,A,C},{5,7,D},{0,5,B},{3,7,A},
{3,4,D},{6,A,D},{4,5,A},{0,C,D},{1,A,B},{1,7,C},{0,2,7},{0, 8,A},{3,5,C},
```

$\{1,2, D\},\{2,3, B\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{1,5,6\},\{2,4,6\}$
$\{4, B, C\},\{0,7, C\},\{2, C, D\},\{1,8, D\},\{4,8, A\},\{1, A, B\},\{5,7, D\},\{0,5, A\},\{0,4, D\}$,
$\{2,5, B\},\{2,3,8\},\{3,7, B\},\{0,8, B\},\{1,3, C\},\{2,7, A\},\{6, A, C\},\{5,8, C\},\{3, A, D\}$,
$\{6, B, D\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{0,5,7\},\{2,4,6\}$
$\{1,3, D\},\{4, A, B\},\{5,6, D\},\{6, A, C\},\{0,4, C\},\{2,3, B\},\{2,7, A\},\{0,8, B\},\{4,8, D\}$,
$\{2, C, D\},\{3,8, A\},\{7, B, D\},\{1,6, B\},\{1,8, C\},\{3,7, C\},\{5, B, C\},\{0, A, D\},\{1,5, A\}$
(ii) See the Appendix.
(iii) The five exceptional values not in Corollary 15 are given in the Appendix.
(iv) Using part (iii) of Corollary 15,

$$
I(27,31) \supseteq I_{3 \subset 7}(27,31)+I(3,7) \supseteq[0,101] \backslash\{98,99\} .
$$

The remaining two intersection values (not required for the recursion which follows) are given by computer construction in the technical report [?].
(v) From the Latin square construction in $[7]$ for $\max I(33,37)$, we have similarly that

$$
I(33,37) \supseteq J+3 * I_{3 \subset 3}(13,13)+I(3,7)=J+([0,76] \backslash\{73,74\}),
$$

where $J=I\left(\operatorname{LLS}\left(2^{5}\right)\right)$. With row swaps as in the proof of Corollary 15 , we have $J \supseteq\{0,16,32, \ldots, 80\}$. So $I(33,37) \supseteq[0,156] \backslash\{153,154\}$. The remaining two cases (not required for the recursion which follows) are presented in the technical report [?].
(vi) Consider two 3 -GDDs of type $12^{3} 6^{1}$ (which have 216 blocks), together with three and seven extra points. As in Theorem 7, we can fill the groups of size 12 and extra points with $\operatorname{ISTS}(15 ; 3)$ and $\operatorname{ISTS}(19 ; 7)$, respectively. On the group of size 6 , we place $\operatorname{STS}(9)$ and $\operatorname{STS}(13)$, filling the holes on the extra points. Therefore, by parts (i) and (ii),

$$
I(45,49) \supseteq 216+3 *[0,26]+[0,8]=[216,302] .
$$

This, taken with Corollary 15, completes the proof.
Lemma 17. For $t \geq 3, I(12 t+3,12 t+7)=[0, b-8 t]$.

Proof. We apply the main construction with $s=3$ and 3 -GDD of type $12^{t}$ :

$$
\begin{aligned}
I(12 t+3,12 t+7) \supseteq & I\left(12^{t}\right)+t * I_{3 \subset 7}(15,19)+I(3,7) \\
= & ([0,24 t(t-1)] \backslash(\{24 t(t-1)\}-\{1,2,3,5\})) \\
& +[0,26 t]+\{0,1\} \\
= & {\left[0,24 t^{2}+2 t+1\right]=[0, b-8 t] }
\end{aligned}
$$

Lemma 18. $I(12 t+21,12 t+25)=[0, b-8 t-12]$.
Proof. First note that the cases $t=0,1,2$ are handled by parts (iii), (v), and (vi) of Lemma 16, respectively. We split the proof for $t \geq 3$ into two cases. First, suppose $t \not \equiv 2(\bmod 3)$. Apply the main construction with $s=3$ and 3 -GDD of type $12^{t} 18^{1}$, using Corollary 9:

$$
\begin{aligned}
I(12 t+21,12 t+25) & \supseteq \quad I\left(12^{t} 18^{1}\right)+t * I_{3 \subset 7}(15,19)+I_{3 \subset 7}(21,25)+I(3,7) \\
\supseteq & \{0,3,6, \ldots, 24 t(t+2)-6,24 t(t+2)\} \\
& +[0,26 t]+[0,57]+\{0,1\} \\
& =\left[0,24 t^{2}+74 t+58\right]=[0, b-8 t-12] .
\end{aligned}
$$

Now if $t \equiv 2(\bmod 3)$, let $t^{\prime}=2(t-2) / 3+1$. Observe $t^{\prime} \geq 3$ since in this case $t \geq 5$. Use 3-GDD of type $18^{t^{\prime}} 24^{1}$ to get the same result as above.

Combining the previous two results, the case $v=u+4$ is now complete.
Theorem 19. $I(6 t+3,6 t+7)=[0, b-4 t]$.

## $5 \quad v=2 u-1$ and $2 u-3$

We begin with a result analogous to Theorem 7. The proof is similar, but relies on an easy group doubling construction for 3-GDDs: that is, any 3-GDD of type $g_{1} \cdots g_{t}$ embeds in a 3 -GDD of type $\left(2 g_{1}\right) \cdots\left(2 g_{t}\right)$.

Theorem 20. Suppose there is a $3-G D D$ of type $T=g_{1} \cdots g_{t}$ and let $s<g_{i}$ for every $i$ be such that $s, g_{i}+s$, and $2 g_{i}+s$ are admissible for $i=1, \ldots, t$. Then

$$
I\left(\Sigma_{i} g_{i}+s, 2 \Sigma_{i} g_{i}+s\right) \supseteq I(3 \mathrm{GDD}(T))+\sum_{i} I_{s \subset s}\left(g_{i}+s, 2 g_{i}+s\right)+I(s, s)
$$

Remark 21. We could in fact state this more generally with $2 g_{i}$ replaced by $h_{i}$, where $h_{i} \geq 2 g_{i}$. However, as noted in [7], such a construction achieves the upper bound in Lemma 2 only when $h_{i}=2 g_{i}$.

## Lemma 22.

(i) $I(7,13)=[0,5]$, and
(ii) $I(13,25)=[0,22]$.

Proof. (i) The following $\operatorname{STS}(7)$ intersect the $\operatorname{STS}(13)$ given in Lemma 11 in $0,1, \ldots, 5$ blocks, respectively.

$$
\begin{aligned}
& \{0,1,3\},\{0,2,4\},\{0,5,8\},\{1,2,5\},\{1,4,8\},\{2,3,8\},\{3,4,5\} \\
& \{0,1,3\},\{0,2,4\},\{0,5,6\},\{1,2,5\},\{1,4,6\},\{2,3,6\},\{3,4,5\} \\
& \{0,1,2\},\{0,3,6\},\{0,4,5\},\{1,3,5\},\{1,4,6\},\{2,3,4\},\{2,5,6\} \\
& \{0,1,2\},\{0,3,4\},\{0,5,6\},\{1,3,6\},\{1,4,5\},\{2,3,5\},\{2,4,6\} \\
& \{0,1,2\},\{0,3,4\},\{0,5,8\},\{1,3,5\},\{1,4,8\},\{2,3,8\},\{2,4,5\} \\
& \{0,1,2\},\{0,3,4\},\{0,5,6\},\{1,3,5\},\{1,4,6\},\{2,3,6\},\{2,4,5\}
\end{aligned}
$$

(ii) A computer construction is given in the technical report [?]. We note that this case is not used in the recursion.

Theorem 23. $I(6 t+1,12 t+1)=[0, b-2 t]$.
Proof. For $t=1,2$, the result follows from Lemma 22. For $t \geq 3$, apply Theorem 20 with 3 -GDD of type $6^{t}$ and $s=1$. We have by Lemma 5

$$
\begin{aligned}
I(6 t+1,12 t+1) & \supseteq I\left(3 \operatorname{GDD}\left(6^{t}\right)\right)+\sum_{i=1}^{t} I_{1 \subset 1}(7,13)+I(1,1) \\
& =([0,6 t(t-1)] \backslash(\{6 t(t-1)\}-\{1,2,3,5\}))+t *[0,5] \\
& =[0, b-2 t] .
\end{aligned}
$$

But from Lemma $2, I(6 t+1,12 t+1) \subseteq[0, b-2 t]$. Hence the result follows.

## Lemma 24.

(i) $I(9,15)=[0,8]$, and
(ii) $I(15,27)=[0,27]$.

Proof. (i) The following $\operatorname{STS}(15)$ have intersections $0, \ldots, 8$ with the $\operatorname{STS}(9)$ given in Lemma 16.
$\{4,7, F\},\{4,5, C\},\{5, B, F\},\{2, B, E\},\{3,5, E\},\{0,3, F\},\{4,6,8\},\{2, C, D\},\{1,5,7\}$, $\{6, E, F\},\{2,3,8\},\{4, D, E\},\{7,8, E\},\{6,7, D\},\{A, D, F\},\{0, A, E\},\{1,6, B\},\{0,5, D\}$, $\{3,6, A\},\{5,8, A\},\{0,6, C\},\{2,7, A\},\{0,1,8\},\{1,4, A\},\{8, B, D\},\{1,2, F\},\{A, B, C\}$, $\{2,5,6\},\{3,7, C\},\{1,3, D\},\{3,4, B\},\{8, C, F\},\{1, C, E\},\{0,7, B\},\{0,2,4\}$
$\{2,4,6\}$
$\{0,8, E\},\{3, A, C\},\{1,3, F\},\{2,7, D\},\{6,7, A\},\{0,1,6\},\{6, C, E\},\{1,4, C\},\{1, B, D\}$,
$\{0, C, D\},\{2,5, C\},\{5,7, B\},\{A, D, F\},\{3, D, E\},\{B, C, F\},\{5,6, F\},\{5, A, E\},\{1,7, E\}$,
$\{7,8, C\},\{0,7, F\},\{4, B, E\},\{0,4, A\},\{0,3,5\},\{4,8, F\},\{3,4,7\},\{1,5,8\},\{2,3,8\}$,
$\{6,8, D\},\{2, E, F\},\{0,2, B\},\{4,5, D\},\{1,2, A\},\{8, A, B\},\{3,6, B\}$
$\{0,1,2\},\{1,5,6\}$
$\{1, C, F\},\{8, A, F\},\{2, B, E\},\{0,3,8\},\{2,6,8\},\{3, A, C\},\{1,4,8\},\{8, B, D\},\{1, D, E\}$, $\{3,6, D\},\{2, A, D\},\{6,7, B\},\{3,5, E\},\{5,7,8\},\{0,7, D\},\{4, C, D\},\{1,3,7\},\{6, E, F\}$, $\{7, A, E\},\{2,4,5\},\{0, B, F\},\{0,6, C\},\{0,4, E\},\{1, A, B\},\{2,3, F\},\{3,4, B\},\{4,6, A\}$, $\{2,7, C\},\{8, C, E\},\{0,5, A\},\{4,7, F\},\{5, B, C\},\{5, D, F\}$
$\{0,5,7\},\{3,4,5\},\{2,3,7\}$
$\{7,8, E\},\{2,5, D\},\{1,2,6\},\{6, D, F\},\{4,8, F\},\{0,6, A\},\{5,6, C\},\{B, D, E\},\{5, B, F\}$, $\{0,4, B\},\{7, A, F\},\{4,6, E\},\{3, C, F\},\{4,7, D\},\{2, A, B\},\{1,7, C\},\{0,2,8\},\{0, C, D\}$,
$\{0,1, F\},\{1,8, D\},\{5,8, A\},\{6,7, B\},\{2,4, C\},\{3, A, D\},\{1,3, B\},\{1,5, E\},\{A, C, E\}$, $\{2, E, F\},\{1,4, A\},\{0,3, E\},\{8, B, C\},\{3,6,8\}$

```
{0,5,7},{0,4,8},{6,7,8},{0,3,6}
{B,C,D},{1,4,B},{4,C,E},{0,E,F},{2,4,7},{2,6,D},{4,6,F},{0,2,B},{6,A,E},
{5,6,B},{0,A,C},{3,8,C},{5,D,F},{4,5,A},{1,7,A},{1,2,E},{1,3,F},{A,B,F},
{1,5,8},{0,1,D},{8,A,D},{2,3,A},{1,6,C},{3,4,D},{2,5,C},{7,C,F},{3,5,E},
{2,8,F},{3,7,B},{7,D,E},{8,B,E}
{0,5,7},{1,5,6},{1,3,8},{0,3,6},{0,4,8}
{4,6,A},{6,B,D},{7,8,C},{0,E,F},{0,2,B},{4,5,C},{0,1,C},{2,3,5},{1,4,B},
{4,7,E},{5,B,F},{3,A,E},{8,B,E},{0,A,D},{3,C,F},{2,4,F},{8,D,F},{1,A,F},
{A,B,C},{1,2,E},{2,C,D},{3,4,D},{6,C,E},{5,8,A},{1,7,D},{5,D,E},{3,7,B},
{2,7,A},{2,6,8},{6,7,F}
{2,4,6},{2,3,7},{0,4,8},{1,5,6},{0,5,7},{2,5,8}
{2,C,F},{3,4,A},{2,A,E},{3,5,D},{1,7,F},{4,7,E},{1,8,A},{4,B,D},{4,5,F},
{5,C,E},{0,A,F},{8,E,F},{3,6,E},{0,2,B},{0,6,C},{6,7,A},{3,B,F},{3,8,C},
{5,A,B},{6,D,F},{0,D,E},{1,2,D},{1,B,E},{0,1,3},{7,B,C},{1,4,C},{6,8,B},
{A,C,D},{7,8,D}
{1,5,6},{3,4,5},{2,4,6},{0,3,6},{2,3,7},{2,5,8},{0,4,8}
{3,8,D},{0,1,7},{6,7,F},{1,4,A},{5,7,D},{4,B,C},{6,C,E},{1,8,F},{0,2,C},
{4,D,F},{2,B,F},{5,A,C},{7,8,C},{0,D,E},{1,3,B},{5,E,F},{6,B,D},{7,A,B},
{6,8,A},{1,C,D},{4,7,E},{3,A,E},{0,5,B},{3,C,F},{1,2,E},{8,B,E},{0,A,F},
{2,A,D}
{0,1,2},{3,4,5},{6,7,8},{1,3,8},{1,4,7},{2,5,8},{0,4,8},{0,5,7}
{7,B,E},{8,D,E},{2,7,A},{1,D,F},{7,C,D},{0,3,D},{2,3,E},{1,A,E},{0,C,E},
{0,B,F},{1,6,C},{0,6,A},{1,5,B},{4,6,D},{5,6,E},{2,6,F},{3,7,F},{2,B,D},
{8,B,C},{3,6,B},{4,A,B},{3,A,C},{2,4,C},{4,E,F},{8,A,F},{5,A,D},{5,C,F}
```

(ii) A computer construction is given in the technical report [?]. We note that this case is not used in the recursion.

Theorem 25. For $t \geq 1, I(6 t+3,12 t+3)=[0, b-4 t]$.
Proof. For $t=1,2$, the result follows from Lemma 24. For $t \geq 3$, apply Theorem 20 with 3 -GDD of type $6^{t}$ and $s=3$. The calculation proceeds

$$
\begin{aligned}
& I(6 t+3,12 t+3) \supseteq I\left(3 \mathrm{GDD}\left(6^{t}\right)\right)+\sum_{i=1}^{t} I_{3 \subset 3}(9,15)+I(3,3) \\
& \quad=([0,6 t(t-1)] \backslash(\{6 t(t-1)\}-\{1,2,3,5\}))+t *[0,7]+\{1\} \\
& =[1, b-4 t]
\end{aligned}
$$

where $I_{3 \subset 3}(9,15)=[0,7]$ follows from part (i) of Lemma 24. Moreover, $0 \in$ $I(6 t+3,12 t+3)$ by Lemma 3. ¿From Lemma $2, I(6 t+3,12 t+3) \subseteq[0, b-4 t]$. Hence the result follows.
$6 \quad v \geq 2 u+1$
An important family of regular graphs are the circulant graphs. Recall that a graph with $n$ vertices (say $\mathbb{Z}_{n}$ ) is circulant if there are nonzero distances $m_{1}, \ldots, m_{s}$ such that $x y$ is an edge if and only if $x-y= \pm m_{i}$ for some $i$. The order of such an edge is $n / \operatorname{gcd}\left(n, m_{i}\right)$.

The following is an easy consequence of Theorem 1 and the Doyen-Wilson theorem, [5], which states that any $\operatorname{STS}(u)$ can be embedded in an $\operatorname{STS}(v)$ provided $v \geq 2 u+1$.

Lemma 26. If $v \geq 2 u+1$, then

$$
I(u, v) \supseteq I(u, u)= \begin{cases}\{0,1,2,3,4,6,12\} & \text { if } v=9 \\ {[0, b] \backslash\{b-1, b-2, b-3, b-5\}} & \text { otherwise }\end{cases}
$$

Proof. Let $(U, \mathcal{B})$ be an $\operatorname{STS}(u)$. Take an $\operatorname{STS}\left(U, \mathcal{B}^{\prime}\right)$ with $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right| \in I(u, u)$, and embed in an $\operatorname{STS}(v)$.

For convenience, we also call upon a recent result on embeddings of PTS into STS.

Theorem 27. ([1]) Any PTS(u) embeds in an $S T S(v)$, provided $v \geq 2 u+1$ is admissible.

With these tools, we can now prove the following theorem.
Theorem 28. If $v \geq 2 u+1$, then $I(u, v)=[0, b]$.
Proof. The result is clear when $u=3$, so we assume $u \geq 7$. By Lemma 26, it suffices to show $\{b-1, b-2, b-3, b-5\} \subset I(u, v)$ for $u \neq 9$ and additionally that $\{b-4, b-7\} \subset I(9, v)$.

First, assume $u=7$ (respectively 9 ). Let $(U, \mathcal{B})$ be an $\operatorname{STS}(u)$. For $r=$ $1,2,3,5$ (respectively $r=1,2,3,4,5,7$ ), we note that it is possible to remove $r$ triples $B_{1}, \ldots, B_{r}$ from $\mathcal{B}$ such that there exist pairs $\left\{x_{i}, y_{i}\right\} \subset B_{i}(i=1, \ldots, r)$ with the graph on vertices $U$ formed by these pairs being 2-edge-colourable. Suppose $\kappa:\{1, \ldots, r\} \rightarrow\left\{a_{1}, a_{2}\right\}$ gives such a colouring. Now form a $\operatorname{PTS}(u+2)$ on $U \cup\left\{a_{1}, a_{2}\right\}$ with triples

$$
\mathcal{B} \backslash\left\{B_{1}, \ldots, B_{r}\right\} \cup\left\{\left\{\kappa(i), x_{i}, y_{i}\right\}: i=, \ldots, r\right\} .
$$

By Theorem 27, this PTS embeds into an $\operatorname{STS}(v)$, say ( $V, \mathcal{B}^{\prime}$ ) whenever $v \geq$ $2 u+5$; that is, for $v \geq 19$ (respectively $v \geq 25$ ). Because one pair from each $B_{i}$ is covered in the PTS, no $B_{i}$ can belong to $\mathcal{B}^{\prime}$. Thus we have $\left|\mathcal{B} \backslash \mathcal{B}^{\prime}\right|=r$, i.e. $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=b-r$. For $u=7, v \leq 19$ and $u=9, v \leq 25$, we have generated STS with the required intersections on computer and they are in the technical report [?]. This completes the proof for $u \leq 9$.

Suppose now that $u \geq 13$. We claim there exists a $\operatorname{PTS}(v-u)$ whose leave $L$ admits a 1 -factorization into $u$ factors, and contains a subgraph isomorphic
to three triangles joined at a vertex. In fact, Stern and Lenz show in [10] that there exists such a PTS whose leave $L$ is circulant and contains a full orbit of triangles, provided $u \geq 13$. In each of the cases of their proof (except the first, which treats the special case $u=9, v=19$ ) at least one difference triple is omitted from the cyclic PTS. Suppose $(x+i, y+i, z+i)(\bmod v-u)$ is an orbit of triangles in $L$. Then $(x, y, z),(x, 2 x-y, z+x-y)$, and $(x, 2 x-z, y+x-z)$ are three triangles in $L$ meeting our requirement. Now, as in [10], we may complete to an $\operatorname{STS}(v)$ missing a sub-STS $(u)$ by joining 1-factors of $L$ to $u$ new points. The six edges incident with $x$ in the triangles above must be joined to six distinct points. Among the triples formed are $T_{1} \cup T_{2} \cup T_{3}$, where

$$
T_{j}=\left\{\left\{x, y_{j}, a_{j}\right\},\left\{x, z_{j}, b_{j}\right\},\left\{y_{j}, z_{j}, c_{j}\right\}\right\}, \quad j=1,2,3
$$

and the $a_{j}$ and $b_{j}$ are all distinct. These nine triples are distinct: a pair from any group are clearly distinct for different $j$, the first group is disjoint from the second because the $a_{j}, b_{j}$ are distinct, and the first and second groups are disjoint from the third because $x \notin\{y, z, 2 x-y, 2 x-z, y+x-z, z+x-y\}$. Now place an $\operatorname{STS}(u)$, say $(U, \mathcal{B})$, on the $u$ new points and align the 3 -subsets $\left\{a_{j}, b_{j}, c_{j}\right\}, j=1,2,3$, with triples of $S$. This is possible because all possible configurations for three blocks exist in any STS of order at least 13 (see [8]). For $r=1,2,3$, an $\operatorname{STS}(v)\left(V, \mathcal{B}^{\prime}\right)$, which intersects $S$ in exactly $b-r$ triples, is formed by exchanging $T_{j} \cup\left\{\left\{a_{j}, b_{j}, c_{j}\right\}\right\}$ with

$$
\left\{\left\{x, a_{j}, b_{j}\right\},\left\{y_{j}, b_{j}, c_{j}\right\},\left\{z_{j}, a_{j}, c_{j}\right\},\left\{x, y_{j}, z_{j}\right\}\right\}
$$

for $1 \leq j \leq h$. Finally, take an $\operatorname{STS}(u)(U, \mathcal{C})$ with $|\mathcal{B} \cap \mathcal{C}|=b-4$ blocks. Align $\mathcal{C}$ on the new points so that, say, $\left\{a_{1}, b_{1}, c_{1}\right\} \in \mathcal{B} \cap \mathcal{C}$. We destroy this triple via an exchange as above to create an $\operatorname{STS}(v)$ agreeing with $\mathcal{B}$ in $b-5$ blocks.

## 7 Conclusion and further work

For convenience we summarize the results of [9] and of this paper in a single theorem which is a statement of current knowledge.

Theorem 29. For $u \leq v$, let $I(u, v)$ be the set of all $x$ such that there exists $S T S(u)$ and $S T S(v)$, say $(U, B)$ and $\left(V, B^{\prime}\right)$ with $U \subseteq V$ and $\left|\mathcal{B} \cap \mathcal{B}^{\prime}\right|=x$. Let $b=u(u-1) / 6$. Then
(i)

$$
I(u, u)= \begin{cases}\{0,1,2,3,4,6,12\} & \text { if } u=9 \\ {[0, b] \backslash\{b-1, b-2, b-3, b-5\}} & \text { otherwise }\end{cases}
$$

(ii)

$$
I(u, u+2)= \begin{cases}\{0,1,2,4\}=[0, b-(u-1) / 2] \backslash\{3\} & \text { if } u=7 \\ {[0,19]=[0, b-(u-1) / 2] \backslash\{20\}} & \text { if } u=13 \\ {[0, b-(u-1) / 2]} & \text { otherwise }\end{cases}
$$

(iii) $I(u, v)=[0, b-(v-u)(2 u+1-v) / 6]$ if $v=u+4,2 u-3$ or $2 u-1$.
(iv) $I(u, v)=[0, b]$ if $v \geq 2 u+1$.

This leaves the spectrum of $I(u, v), u+6 \leq v \leq 2 u-5$ unresolved but given the results above, we think it is quite reasonable to conjecture:

Conjecture 30. For $u+4 \leq v \leq 2 u-1, I(u, v)=[0, b-(v-u)(2 u+1-v) / 6]$.
From [7], we know $b-(v-u)(2 u+1-v) / 6$ is the maximum possible value of $I(u, v)$ when $u \leq v \leq 2 u+1$ and this bound is achieved for various cases (when $v=u+6, u+8, u+10,2 u-5)$ not treated in this paper. By Theorem 3, the minimum possible value 0 of $I(u, v)$ is also achieved in these cases.

Finally, it should also be noted that while many of the same computational and recursive techniques employed here for $I(u, v)$ and in [7] for the maximum value of $I(u, v)$ can be extended to other values of $v$ near $u$ (as in part (v) of Lemma 16) or $2 u+1$, we are near the limit of both the reasonable computation limit and size of output for the base cases and exceptional values using these techniques. Further, we presently lack a good understanding of how to determine $I(u, v)$ in general and even for the restriction to a given "fiber" $\frac{u}{v} \rightarrow k$, where $1<k<2$.

## Appendix: Computer generated objects needed for the recursive constructions Appendix (a) $\quad I_{3 \subset 7}(15,19)$

For $I_{3 \subset 7}(15,19)$, we first give four pairs of $\operatorname{ISTS}(15 ; 3)$ and $\operatorname{ISTS}(19 ; 7)$ with intersections 23, 24, 25, 26. Moreover, various block-disjoint Pasch configurations are shown to be contained in the intersections. By applying zero or more Pasch trades to each $\operatorname{ISTS}(15 ; 3)$, we see that $I_{3 \subset 7}(15,19) \supseteq\{4\} \cup[6,26]$.

Let $U=\{0, \ldots, 15\}, H=\{0,1,2\}, V=U \cup\{A, B, C, D\}$, and $H^{\prime}=H \cup$ $\{A, B, C, D\}$. Below, we give block sets for $\operatorname{ISTS}(15 ; 3)(U, H, \mathcal{B})$ and $\operatorname{ISTS}(19 ; 7)$ $\left(V, H^{\prime}, \mathcal{B}^{\prime}\right)$ having intersections $23,24,25,26$. First, $\mathcal{B} \backslash \mathcal{B}^{\prime}$ is given, then $\mathcal{B} \cap \mathcal{B}^{\prime}$ with Pasch configurations noted, then $\mathcal{B}^{\prime} \backslash \mathcal{B}$.
a-i: $\quad 7,11,15,19,23 \in I_{3 \subset 7}(15,19)$
$\{5,8,14\},\{4,7,12\},\{6,8,13\},\{6,10,11\},\{3,9,13\},\{4,10,13\},\{4,9,14\},\{5,7,13\},\{3,7,11\}$, $\{4,8,11\},\{5,10,12\}$

Pasch 1: $\{0,5,6\},\{0,11,12\},\{5,9,11\},\{6,9,12\}$ Pasch 2: $\{1,3,5\},\{1,4,6\},\{2,3,6\},\{2,4,5\}$
Pasch 3: $\{2,8,9\},\{2,7,10\},\{0,9,10\},\{0,7,8\}$ Pasch 4: $\{2,12,13\},\{2,11,14\},\{1,12,14\},\{1,11,13\}$
$\{0,1,2\},\{3,8,12\},\{0,3,4\},\{1,8,10\},\{1,7,9\},\{0,13,14\},\{3,10,14\},\{6,7,14\}$
$\{4,10,11\},\{5,8,13\},\{4,7,13\},\{6,8, A\},\{4,12, C\},\{10,12, D\},\{6,13, D\},\{4,9, D\},\{5,10, B\}$,
$\{9,13, C\},\{6,10, C\},\{6,11, B\},\{5,7, D\},\{7,12, B\},\{4,8, B\},\{5,12, A\},\{3,7, C\},\{8,11, C\}$, $\{7,11, A\},\{3,9, A\},\{5,14, C\},\{9,14, B\},\{3,13, B\},\{3,11, D\},\{10,13, A\},\{8,14, D\},\{4,14, A\}$
a-ii: $\quad 4,8,12,16,20,24 \in I_{3 \subset 7}(15,19)$
$\{5,7,13\},\{4,7,12\},\{5,10,12\},\{6,9,12\},\{6,7,14\},\{4,8,11\},\{5,8,14\},\{3,10,14\},\{6,10,11\}$, $\{3,9,13\}$

Pasch 1: $\{0,7,8\{0,11,12\},\{3,7,11\},\{3,8,12\}$ Pasch 2: $\{0,9,10\},\{0,13,14\},\{4,9,14\},\{4,10,13\}$
Pasch 3: $\{1,7,9\},\{1,8,10\},\{2,7,10\},\{2,8,9\}$ Pasch 4: $\{2,4,5\},\{2,3,6\},\{1,4,6\},\{1,3,5\}$
Pasch 5: $\{2,12,13\},\{2,11,14\},\{1,12,14\},\{1,11,13\}$
$\{0,3,4\},\{0,5,6\},\{5,9,11\},\{6,8,13\}$
$\{5,7,14\},\{6,10,12\},\{4,11, B\},\{9,12, B\},\{6,11, A\},\{6,7, B\},\{10,11, D\},\{5,12, C\},\{6,9, C\}$, $\{5,8, D\},\{3,10, B\},\{9,13, A\},\{4,12, D\},\{4,7, C\},\{5,13, B\},\{8,11, C\},\{10,14, C\},\{3,14, A\}$ $\{7,13, D\},\{3,13, C\},\{6,14, D\},\{5,10, A\},\{4,8, A\},\{7,12, A\},\{8,14, B\},\{3,9, D\}$
a-iii: $\quad 9,13,17,21,25 \in I_{3 \subset 7}(15,19)$
$\{5,11,14\},\{3,10,11\},\{4,9,14\},\{5,8,13\},\{4,10,12\},\{6,7,12\},\{3,8,9\},\{6,10,13\},\{4,7,13\}$
Pasch 1: $\{0,5,6\},\{0,11,12\},\{5,9,12\},\{6,9,11\}$ Pasch 2: $\{1,8,10\},\{1,12,14\},\{2,8,12\},\{2,10,14\}$
Pasch 3: $\{2,4,5\},\{2,3,6\},\{1,4,6\},\{1,3,5\} \quad$ Pasch 4: $\{2,7,11\},\{2,9,13\},\{1,11,13\},\{1,7,9\}$ $\{4,8,11\},\{0,3,4\},\{0,7,8\},\{5,7,10\},\{0,9,10\},\{3,12,13\},\{0,13,14\},\{6,8,14\},\{3,7,14\}$
$\{4,10,13\},\{9,14, A\},\{10,12, D\},\{10,11, A\},\{11,14, D\},\{6,12, B\},\{7,12, C\},\{4,14, B\}$, $\{4,9, C\},\{8,9, B\},\{3,8, A\},\{7,13, B\},\{3,11, C\},\{6,10, C\},\{5,13, A\},\{3,9, D\},\{6,7, A\}$, $\{5,11, B\},\{3,10, B\},\{4,12, A\},\{5,8, D\},\{5,14, C\},\{4,7, D\},\{6,13, D\},\{8,13, C\}$
a-iv: $\quad 6,10,14,18,22,26 \in I_{3 \subset 7}(15,19)$
$\{3,9,13\},\{5,7,14\},\{3,8,12\},\{4,8,11\},\{6,7,13\},\{5,10,11\},\{6,10,12\},\{4,9,14\}$
Pasch 1: $\{0,3,4\},\{0,11,12\},\{3,7,11\},\{4,7,12\}$ Pasch 2: $\{0,5,6\},\{0,13,14\},\{5,8,13\},\{6,8,14\}$
Pasch 3: $\{0,7,8\},\{0,9,10\},\{2,7,10\},\{2,8,9\}$ Pasch 4: $\{2,4,5\},\{2,3,6\},\{1,4,6\},\{1,3,5\}$
Pasch 5: $\{2,12,13\},\{2,11,14\},\{1,12,14\},\{1,11,13\}$
$\{3,10,14\},\{1,7,9\},\{4,10,13\},\{5,9,12\},\{6,9,11\},\{1,8,10\}$
$\{9,14, D\},\{3,9, C\},\{4,11, D\},\{5,11, A\},\{7,13, C\},\{7,14, A\},\{4,14, B\},\{5,14, C\},\{8,11, B\}$, $\{6,13, D\},\{10,12, D\},\{10,11, C\},\{5,10, B\},\{3,12, B\},\{6,12, C\},\{4,8, C\},\{5,7, D\},\{3,8, D\}$, $\{4,9, A\},\{8,12, A\},\{6,10, A\},\{6,7, B\},\{9,13, B\},\{3,13, A\}$

To complete the interval $[0,26]$, five further examples are required. Consider the following $\operatorname{ISTS}(15 ; 3)$.

```
{1, 12,14},{4, 8, 11},{0,3,4},{2,3,6},{4, 9,14},{0,5,6},{2,4,5},{4,10,13},{0,7,8},
{2,7,10},{5,7,13},{0,9,10},{2,8,9},{5,8,14},{0,11,12},{2,11,14},{5,9,11},{0,13,14},
{2,12,13},{5,10,12},{1,3,5},{3,7,11},{6,7,14},{1,4,6},{3,8,12},{6,8,13},{1,7,9},
{3,9,13},{6,9,12},{1,8,10},{3,10,14}, {6,10,11}, {1,11,13}, {4,7,12}
```

The following $\operatorname{ISTS}(19 ; 7)$ have intersections $0,1,2,3,5$ with this system.
$\{10,14, C\},\{3,7, D\},\{11,12, A\},\{4,6, D\},\{9,13, B\},\{2,9,14\},\{12,13, C\},\{1,10,13\}$, $\{3,8, B\},\{2,6,13\},\{1,3,14\},\{3,9, C\},\{0,7,9\},\{6,10, B\},\{1,9,12\},\{2,7,12\},\{2,5,10\}$, $\{8,10, D\},\{11,14, B\},\{3,5,13\},\{7,13, A\},\{8,13,14\},\{8,9, A\},\{4,12, B\},\{1,6,7\},\{4,11, C\}$, $\{3,6,12\},\{1,4,8\},\{0,6,8\},\{2,8,11\},\{6,14, A\},\{7,10,11\},\{11,13, D\},\{5,9, D\},\{5,7, B\}$, $\{0,4,13\},\{0,3,11\},\{5,8,12\},\{4,5, A\},\{12,14, D\},\{1,5,11\},\{2,3,4\},\{0,10,12\},\{0,5,14\}$, $\{7,8, C\},\{6,9,11\},\{5,6, C\},\{3,10, A\},\{4,7,14\},\{4,9,10\}$
$\{0,9,10\}\{4,6,14\},\{1,5,10\},\{3,4, C\},\{8,14, A\},\{2,10,13\},\{0,6,8\},\{4,10, D\},\{0,3,5\}$, $\{11,12, A\},\{1,7,14\},\{2,5,14\},\{8,9, C\},\{7,9, D\},\{4,9, A\},\{5,13, A\},\{3,6, B\},\{0,12,14\}$, $\{5,9, B\},\{3,11, D\},\{0,7,11\},\{7,10,12\},\{4,12, B\},\{11,14, C\},\{1,6,13\},\{13,14, D\}$,
$\{3,10, A\},\{5,12, C\},\{7,13, C\},\{8,10,11\},\{1,3,12\},\{4,5,7\},\{3,8,13\},\{1,9,11\},\{1,4,8\}$, $\{5,6,11\},\{11,13, B\},\{2,6,9\},\{9,12,13\},\{2,3,7\},\{6,12, D\},\{2,8,12\},\{2,4,11\},\{7,8, B\}$, $\{6,7, A\},\{5,8, D\},\{3,9,14\},\{0,4,13\},\{6,10, C\},\{10,14, B\}$
$\{1,3,5\},\{2,4,5\}, \quad\{6,12, A\},\{2,10,13\},\{4,10, A\},\{0,8,12\},\{1,7,12\},\{13,14, D\},\{2,3,7\}$,
$\{0,5,13\},\{2,12,14\},\{1,4,8\},\{9,11, D\},\{3,9, B\},\{6,7, B\},\{4,12, B\},\{4,7,11\},\{3,8, D\}$,
$\{3,6, C\},\{9,12, C\},\{8,10, B\},\{0,7,14\},\{4,6, D\},\{5,6,10\},\{1,9,10\},\{1,6,13\},\{5,12, D\}$,
$\{10,11,12\},\{2,6,9\},\{8,9,13\},\{0,3,10\},\{9,14, A\},\{5,14, B\},\{3,11, A\},\{7,13, A\},\{5,7,9\}$,
$\{3,12,13\},\{0,6,11\},\{2,8,11\},\{4,13, C\},\{7,8, C\},\{10,14, C\},\{5,11, C\},\{0,4,9\}$,
$\{11,13, B\},\{6,8,14\},\{7,10, D\},\{5,8, A\},\{1,11,14\},\{3,4,14\}$
$\{6,8,13\},\{4,9,14\},\{3,7,11\}, \quad\{0,3,10\},\{1,3,6\},\{3,14, B\},\{1,4,11\},\{4,6, C\},\{0,9,11\}$, $\{2,8,11\},\{1,8,9\},\{10,13, C\},\{6,9, A\},\{0,5,12\},\{5,11, A\},\{3,12, C\},\{9,12, B\},\{5,9, C\}$, $\{7,8, C\},\{4,5, D\},\{6,11, D\},\{10,11,12\},\{2,9,13\},\{11,13, B\},\{1,13,14\},\{12,13, D\}$, $\{6,10, B\},\{2,6,12\},\{7,13, A\},\{2,10,14\},\{0,6,7\},\{12,14, A\},\{11,14, C\},\{7,9,10\}$, $\{5,8, B\},\{4,7, B\},\{3,5,13\},\{2,5,7\},\{5,6,14\},\{0,8,14\},\{3,8, A\},\{1,7,12\},\{0,4,13\}$, $\{8,10, D\},\{1,5,10\},\{2,3,4\},\{7,14, D\},\{4,8,12\},\{4,10, A\},\{3,9, D\}$
$\{6,7,14\},\{4,7,12\},\{0,3,4\},\{0,9,10\},\{4,10,13\},\{1,6,10\},\{5,10, A\},\{2,7,8\},\{7,9, A\}$, $\{3,10, B\},\{9,12,13\},\{3,14, A\},\{5,6, C\},\{4,8, A\},\{0,8,14\},\{4,11, B\},\{2,6,11\},\{8,10,11\}$, $\{8,13, C\},\{7,10, D\},\{0,6,12\},\{5,7, B\},\{3,5,11\},\{0,5,13\},\{6,9, B\},\{13,14, B\},\{3,6,8\}$, $\{1,4,5\},\{8,12, B\},\{2,10,14\},\{2,3,13\},\{1,8,9\},\{4,14, C\},\{11,12, A\},\{10,12, C\},\{2,4,9\}$, $\{0,7,11\},\{12,14, D\},\{9,11, C\},\{1,3,12\},\{11,13, D\},\{1,11,14\},\{6,13, A\},\{5,9,14\}$,
$\{5,8, D\},\{2,5,12\},\{1,7,13\},\{3,9, D\},\{3,7, C\},\{4,6, D\}$

## Appendix (b) $\quad I_{3 \subset 7}(21,25)$

We now give direct constructions for $54,55,56 \in I_{3 \subset 7}(21,25)$. The other two cases not already covered in Corollary 15 are handled through Pasch trades.
b-i: $\quad 48,52,56 \in I_{3 \subset 7}(21,25)$
$\{6,15,19\},\{5,10,17\},\{9,12,14\},\{14,16,20\},\{10,11,18\},\{9,11,20\},\{7,9,13\},\{13,15,18\}$, $\{7,8,19\},\{3,8,16\},\{4,6,12\},\{3,4,5\},\{13,17,20\}$

Pasch 1: $\{0,7,18\}\{0,10,19\}\{7,10,20\}\{18,19,20\}$ Pasch 2: $\{5,8,11\}\{2,5,20\}\{8,12,20\}\{2,11,12\}$ $\{7,14,17\},\{2,8,17\},\{0,6,20\},\{9,10,16\},\{4,8,9\},\{1,16,18\},\{0,8,13\},\{1,6,13\},\{10,12,15\}$, $\{3,10,13\},\{11,14,15\},\{4,17,18\},\{1,5,9\},\{5,12,13\},\{3,12,18\},\{5,6,18\},\{1,11,17\}$,
$\{15,16,17\},\{3,9,19\},\{4,7,16\},\{1,3,20\},\{2,10,14\},\{0,4,11\},\{2,4,19\},\{0,12,16\},\{6,8,10\}$, $\{2,13,16\},\{11,13,19\},\{6,11,16\},\{5,16,19\},\{8,14,18\},\{5,7,15\},\{0,3,17\},\{2,9,18\}$, $\{1,7,12\},\{1,4,10\},\{1,8,15\},\{1,14,19\},\{2,3,15\},\{3,7,11\},\{4,15,20\},\{2,6,7\},\{4,13,14\}$, $\{6,9,17\},\{3,6,14\},\{0,9,15\},\{12,17,19\},\{0,5,14\}$
$\{9,13,20\},\{3,5, B\},\{7,9, B\},\{4,5, C\},\{7,13, D\},\{6,19, B\},\{5,17, D\},\{7,8, C\},\{5,10, A\}$, $\{14,16, A\},\{17,20, A\},\{11,20, C\},\{9,11, D\},\{9,12, A\},\{10,17, C\},\{15,18, B\},\{10,18, D\}$, $\{3,8, A\},\{4,6, A\},\{15,19, C\},\{4,12, B\},\{13,17, B\},\{11,18, A\},\{16,20, D\},\{14,20, B\}$, $\{3,4, D\},\{6,15, D\},\{3,16, C\},\{7,19, A\},\{9,14, C\},\{6,12, C\},\{13,15, A\},\{8,19, D\}$, $\{10,11, B\},\{8,16, B\},\{12,14, D\},\{13,18, C\}$
b-ii: $\quad 54 \in I_{3 \subset 7}(21,25)$
$\{14,17,18\},\{5,8,15\},\{3,7,8\},\{7,11,13\},\{10,16,18\},\{14,15,19\},\{3,6,15\},\{13,15,16\}$, $\{12,19,20\},\{4,15,17\},\{3,9,19\},\{5,10,12\},\{4,6,16\},\{8,14,20\},\{4,9,11\}$
$\{1,3,14\},\{0,4,7\},\{3,5,11\},\{0,12,15\},\{10,11,15\},\{4,5,20\},\{6,9,12\},\{6,8,10\},\{8,12,18\}$, $\{6,11,14\},\{1,4,10\},\{1,15,20\},\{2,5,18\},\{6,17,19\},\{0,8,16\},\{9,13,18\},\{0,6,18\}$, $\{2,11,12\},\{3,10,20\},\{1,18,19\},\{2,7,19\},\{1,5,9\},\{1,6,13\},\{1,7,17\},\{0,10,19\},\{7,15,18\}$, $\{7,12,14\},\{5,6,7\},\{2,10,17\},\{0,5,14\},\{11,16,19\},\{1,12,16\},\{11,18,20\},\{4,12,13\}$,
$\{10,13,14\},\{4,8,19\},\{0,9,20\},\{8,9,17\},\{1,8,11\},\{2,3,16\},\{7,9,10\},\{2,8,13\},\{13,17,20\}$, $\{5,13,19\},\{2,6,20\},\{5,16,17\},\{2,9,15\},\{0,3,13\},\{9,14,16\},\{7,16,20\},\{2,4,14\},\{3,4,18\}$, $\{0,11,17\},\{3,12,17\}$
$\{3,15,19\},\{4,15,16\},\{8,14,15\},\{4,9, A\},\{14,20, D\},\{8,20, A\},\{4,11, D\},\{7,8, D\}$, $\{16,18, A\},\{12,19, A\},\{10,18, D\},\{7,11, B\},\{3,8, C\},\{5,10, A\},\{15,17, D\},\{3,9, B\}$, $\{6,16, B\},\{6,15, A\},\{14,18, B\},\{9,11, C\},\{10,16, C\},\{12,20, C\},\{4,6, C\},\{3,7, A\}$, $\{14,19, C\},\{7,13, C\},\{19,20, B\},\{11,13, A\},\{5,12, D\},\{9,19, D\},\{17,18, C\},\{3,6, D\}$,
$\{5,8, B\},\{10,12, B\},\{13,15, B\},\{5,15, C\},\{13,16, D\},\{4,17, B\},\{14,17, A\}$
b-iii: $\quad 55 \in I_{3 \subset 7}(21,25)$
$\{6,11,12\},\{5,7,17\},\{11,14,18\},\{4,7,13\},\{5,9,16\},\{9,18,20\},\{15,19,20\},\{12,13,16\}$, $\{10,11,17\},\{3,8,12\},\{4,15,17\},\{8,11,15\},\{8,14,19\},\{3,6,10\}$
$\{9,10,12\},\{1,5,8\},\{4,8,10\},\{3,16,17\},\{5,10,14\},\{1,6,13\},\{0,16,18\},\{4,5,20\},\{2,10,20\}$, $\{6,7,20\},\{2,5,12\},\{13,17,19\},\{0,13,14\},\{4,11,16\},\{1,16,20\},\{8,17,18\},\{8,9,13\}$,
$\{7,15,16\},\{1,9,11\},\{1,12,19\},\{3,14,20\},\{6,9,19\},\{2,4,19\},\{0,6,17\},\{2,6,18\},\{9,14,15\}$,
$\{0,3,19\},\{11,13,20\},\{6,8,16\},\{0,8,20\},\{7,11,19\},\{1,3,15\},\{10,16,19\},\{12,17,20\}$,
$\{1,7,10\},\{2,3,11\},\{10,13,18\},\{2,13,15\},\{0,4,12\},\{3,7,18\},\{1,4,18\},\{4,6,14\},\{3,5,13\}$,
$\{3,4,9\},\{0,5,11\},\{2,14,16\},\{12,15,18\},\{5,18,19\},\{1,14,17\},\{7,12,14\},\{2,9,17\}$,
$\{5,6,15\},\{0,10,15\},\{0,7,9\},\{2,7,8\}$
$\{8,11,12\},\{11,15,17\},\{3,8, D\},\{5,16, C\},\{3,12, C\},\{8,15, C\},\{15,20, B\},\{5,9, B\}$, $\{10,11, B\},\{11,18, D\},\{7,13, C\},\{3,10, A\},\{9,20, D\},\{4,17, C\},\{4,7, A\},\{15,19, A\}$,
$\{19,20, C\},\{6,12, D\},\{8,14, A\},\{13,16, D\},\{9,16, A\},\{7,17, B\},\{12,16, B\},\{11,14, C\}$, $\{14,18, B\},\{18,20, A\},\{14,19, D\},\{4,15, D\},\{5,17, A\},\{6,11, A\},\{9,18, C\},\{8,19, B\}$, $\{5,7, D\},\{3,6, B\},\{12,13, A\},\{4,13, B\},\{10,17, D\},\{6,10, C\}$

## Part II

## The Computational Results

## A Cases from Lemma 16:

Consider the following PTS(7) appearing in [7].

$$
\beta_{0}: \quad\{\{0,1,2\},\{1,3,4\},\{2,3,5\},\{4,5,6\}\}
$$

This has the property that its underlying graph is 4-edge-colourable with the degree two vertices 0 and 6 incident with disjoint pairs of colours.

In a similar way, we have $\operatorname{PTS}(9) \beta_{2}$ and $\beta_{3}$ whose underlying graphs can be edge-decomposed into 2 and 3 "new" triples, respectively, and a 4-edgecolourable graph $G$. Moreover, exactly six of the vertices of $G$ have degree two, and all six pairs of colours are "missing" at these vertices. We leave it to the reader to check these conditions.

| $\beta_{2}$ | new | $\beta_{3}$ | new |
| :---: | :---: | :---: | :---: |
| $\{0,1,2\}$ |  | $\{0,1,2\}$ |  |
| $\{2,3,4\}$ |  | $\{1,3,4\}$ |  |
| $\{2,5,6\}$ | $\{2,5,7\}$ | $\{1,6,7\}$ | $\{1,2,6\}$ |
| $\{2,7,8\}$ | $\{2,6,8\}$ | $\{2,4,7\}$ | $\{1,4,7\}$ |
| $\{3,5,7\}$ |  | $\{2,5,6\}$ | $\{2,5,7\}$ |
| $\{4,6,8\}$ |  | $\{3,5,7\}$ |  |
|  |  | $\{4,6,8\}$ |  |

Using $\beta_{0}, \beta_{2}$, and $\beta_{3}$, we can show existence of $\operatorname{STS}(u+4)$ intersecting an $\operatorname{STS}(u)$ in max $I(u, u+4)-2$ and max $I(u, u+4)-3$ blocks. In each of the cases which follow, we specify an $\operatorname{STS}(u)$ containing a PTS consisting of linked copies of $\beta_{0}$ (on points $\{0, \ldots, 6\},\{6, \ldots, 12\}$, etc.), followed by a copy of either $\beta_{2}$ or $\beta_{3}$ on $\{u-9, \ldots, u-1\}$. As in Example 12, we remove the triples of this PTS, add back the new triples, join colour classes of edges in the remaining graph $G$ to four new points, and join pairs of the new points to the deficient vertices in $G$.

A(a) (iv): $\max -2, \max -3 \in I(27,31)$

$$
\max -2 \in I(27,31)
$$

PTS: $\{0,1,2\},\{1,3,4\},\{2,3,5\},\{4,5,6\},\{6,7,8\},\{7,9,10\},\{8,9,11\}$, $\{10,11,12\},\{12,13,14\},\{13,15,16\},\{14,15,17\},\{16,17,18\},\{18,19,20\}$, $\{20,21,22\},\{20,23,24\},\{20,25,26\},\{21,23,25\},\{22,24,26\}$
$\{4,12,24\},\{4,19,23\},\{7,16,22\},\{5,12,16\},\{2,18,24\},\{0,6,15\},\{1,9,13\}$,
$\{8,13,19\},\{8,15,21\},\{1,8,23\},\{1,7,20\},\{5,13,20\},\{7,15,18\},\{4,10,26\}$,
$\{2,4,21\},\{2,8,14\},\{8,18,26\},\{3,7,12\},\{2,7,11\},\{3,16,21\},\{6,14,18\}$,
$\{1,5,19\},\{10,15,25\},\{11,16,24\},\{3,9,20\},\{9,12,17\},\{17,21,26\}$,
$\{11,13,21\},\{4,14,16\},\{11,15,20\},\{10,18,23\},\{0,16,20\},\{3,6,13\}$, $\{1,11,18\},\{0,13,26\},\{2,16,23\},\{1,6,16\},\{12,18,22\},\{8,17,24\},\{9,14,22\}$, $\{4,8,20\},\{9,16,25\},\{2,15,26\},\{0,19,25\},\{1,14,21\},\{11,14,19\},\{5,9,24\}$, $\{6,9,21\},\{0,14,24\},\{1,17,25\},\{5,8,22\},\{2,6,10\},\{0,9,18\},\{5,11,17\}$, $\{0,3,8\},\{2,9,19\},\{2,12,20\},\{0,5,10\},\{6,12,23\},\{0,11,22\},\{4,11,25\}$, $\{6,17,20\},\{9,23,26\},\{12,15,19\},\{1,15,24\},\{10,13,24\},\{5,7,26\}$, $\{5,18,21\},\{7,14,23\},\{1,12,26\},\{8,10,16\},\{0,4,7\},\{7,17,19\},\{6,11,26\}$, $\{7,13,25\},\{3,15,22\},\{8,12,25\},\{10,14,20\},\{0,12,21\},\{6,24,25\}$, $\{3,14,26\},\{10,19,21\},\{5,15,23\},\{3,10,17\},\{16,19,26\},\{4,13,18\}$, $\{13,22,23\},\{3,18,25\},\{1,10,22\},\{2,13,17\},\{6,19,22\},\{0,17,23\}$, $\{4,9,15\},\{4,17,22\},\{3,19,24\},\{7,21,24\},\{3,11,23\},\{2,22,25\},\{5,14,25\}$

$$
\max -3 \in I(27,31)
$$

PTS: $\{0,1,2\},\{1,3,4\},\{2,3,5\},\{4,5,6\},\{6,7,8\},\{7,9,10\},\{8,9,11\}$, $\{10,11,12\},\{12,13,14\},\{13,15,16\},\{14,15,17\},\{16,17,18\},\{18,19,20\}$, $\{19,21,22\},\{20,23,24\},\{20,22,25\},\{19,24,25\},\{21,23,25\},\{22,24,26\}$ $\{0,13,20\},\{1,11,19\},\{4,11,24\},\{6,11,23\},\{0,4,26\},\{6,21,26\},\{5,12,19\}$, $\{1,8,16\},\{2,12,21\},\{14,18,21\},\{5,9,13\},\{9,18,24\},\{1,6,18\},\{4,7,19\}$, $\{7,13,18\},\{3,17,19\},\{13,17,25\},\{2,9,17\},\{10,13,26\},\{3,7,16\},\{3,11,22\}$, $\{0,15,18\},\{14,23,26\},\{10,16,22\},\{1,9,26\},\{6,14,22\},\{0,3,14\},\{4,13,23\}$, $\{5,15,23\},\{8,13,22\},\{9,19,23\},\{8,17,23\},\{7,25,26\},\{8,14,24\},\{6,17,24\}$, $\{11,15,21\},\{0,8,25\},\{11,16,26\},\{5,10,21\},\{9,14,16\},\{12,18,23\},\{2,10,24\}$, $\{3,10,23\},\{11,18,25\},\{3,8,21\},\{7,20,21\},\{0,6,12\},\{4,9,22\},\{1,10,15\}$, $\{5,7,22\},\{2,20,26\},\{16,21,24\},\{1,5,20\},\{4,8,15\},\{10,14,19\},\{2,11,13\}$, $\{3,13,24\},\{0,7,11\},\{11,17,20\},\{2,18,22\},\{4,10,18\},\{0,10,17\},\{6,13,19\}$, $\{5,8,18\},\{3,18,26\},\{8,10,20\},\{3,15,25\},\{3,12,20\},\{2,6,15\},\{2,8,19\}$, $\{0,9,21\},\{5,16,25\},\{15,19,26\},\{1,14,25\},\{4,14,20\},\{5,17,26\},\{4,12,16\}$, $\{12,15,22\},\{8,12,26\},\{4,17,21\},\{6,10,25\},\{7,15,24\},\{2,4,25\},\{6,16,20\}$, $\{0,22,23\},\{1,7,23\},\{5,11,14\},\{3,6,9\},\{9,15,20\},\{2,7,14\},\{2,16,23\},\{0,5,24\}$, $\{9,12,25\},\{1,13,21\},\{0,16,19\},\{1,12,24\},\{1,17,22\},\{7,12,17\}$
$\mathbf{A}(\mathbf{b}) \quad(\mathbf{v}): \max -2, \max -3 \in I(33,37)$

$$
\max -2 \in I(33,37)
$$

PTS: $\{0,1,2\},\{1,3,4\},\{2,3,5\},\{4,5,6\},\{6,7,8\},\{7,9,10\},\{8,9,11\}$, $\{10,11,12\},\{12,13,14\},\{13,15,16\},\{14,15,17\},\{16,17,18\},\{18,19,20\}$, $\{19,21,22\},\{20,21,23\},\{22,23,24\},\{24,25,26\},\{26,27,28\},\{26,29,30\}$,
$\{26,31,32\},\{27,29,31\},\{28,30,32\}$
$\{12,21,30\},\{0,17,25\},\{0,4,31\},\{10,13,24\},\{5,21,31\},\{7,13,31\}$, $\{9,12,27\},\{5,14,28\},\{3,17,24\},\{6,9,15\},\{1,6,25\},\{4,21,26\},\{11,18,27\}$, $\{4,23,27\},\{0,6,19\},\{2,20,30\},\{8,25,27\},\{4,12,15\},\{1,11,14\},\{3,12,31\}$, $\{5,24,30\},\{0,9,21\},\{8,18,22\},\{2,17,21\},\{8,24,28\},\{7,14,16\}$,
$\{12,19,32\},\{25,28,31\},\{9,23,31\},\{11,24,31\},\{11,17,28\},\{16,20,26\}$, $\{6,10,18\},\{9,16,30\},\{10,19,30\},\{4,14,30\},\{5,9,26\},\{14,18,21\}$, $\{0,5,10\},\{13,19,29\},\{7,18,26\},\{6,14,24\},\{8,14,32\},\{5,16,19\},\{2,8,13\}$,
$\{1,19,28\},\{6,16,27\},\{0,7,28\},\{1,17,31\},\{3,8,19\},\{15,19,26\},\{6,11,26\}$, $\{14,19,23\},\{5,12,25\},\{13,20,27\},\{21,24,29\},\{5,7,15\},\{1,8,15\}$, $\{9,19,25\},\{5,22,27\},\{10,22,25\},\{2,12,26\},\{2,15,24\},\{4,10,29\}$, $\{0,12,22\},\{7,22,30\},\{3,9,14\},\{4,8,16\},\{3,18,29\},\{3,13,30\},\{4,7,25\}$, $\{10,14,26\},\{16,25,29\},\{2,10,28\},\{1,12,20\},\{2,4,11\},\{7,11,19\}$, $\{8,17,30\},\{16,22,28\},\{3,7,32\},\{7,24,27\},\{8,20,31\},\{5,11,20\}$, $\{6,13,21\},\{17,22,26\},\{4,20,28\},\{3,10,20\},\{4,9,18\},\{9,17,29\}$, $\{8,10,21\},\{0,3,16\},\{11,16,21\},\{2,16,32\},\{2,6,23\},\{6,20,29\},\{0,15,30\}$, $\{15,23,32\},\{2,14,27\},\{4,13,22\},\{1,16,23\},\{21,25,32\},\{13,23,25\}$, $\{4,19,24\},\{3,15,25\},\{7,17,20\},\{9,20,32\},\{0,14,29\},\{1,27,30\}$, $\{7,12,23\},\{5,13,17\},\{6,12,17\},\{6,30,31\},\{2,19,31\},\{14,20,25\}$, $\{12,16,24\},\{14,22,31\},\{8,12,29\},\{1,13,26\},\{6,22,32\},\{10,16,31\}$, $\{1,10,32\},\{15,18,31\},\{10,15,27\},\{5,8,23\},\{3,6,28\},\{11,25,30\}$, $\{1,7,21\},\{0,8,26\},\{3,21,27\},\{9,13,28\},\{18,24,32\},\{3,23,26\},\{2,7,29\}$, $\{3,11,22\},\{17,19,27\},\{15,21,28\},\{2,9,22\},\{10,17,23\},\{11,15,29\}$, $\{23,28,29\},\{11,13,32\},\{2,18,25\},\{0,20,24\},\{15,20,22\},\{1,22,29\}$, $\{18,23,30\},\{5,29,32\},\{1,5,18\},\{12,18,28\},\{0,27,32\},\{1,9,24\}$, $\{4,17,32\},\{0,11,23\},\{0,13,18\}$

$$
\max -3 \in I(33,37)
$$

PTS: $\{0,1,2\},\{1,3,4\},\{2,3,5\},\{4,5,6\},\{6,7,8\},\{7,9,10\},\{8,9,11\}$,
$\{10,11,12\},\{12,13,14\},\{13,15,16\},\{14,15,17\},\{16,17,18\},\{18,19,20\}$, $\{19,21,22\},\{20,21,23\},\{22,23,24\},\{24,25,26\},\{25,27,28\},\{26,29,30\}$, $\{26,28,31\},\{25,30,31\},\{27,31,29\},\{28,30,32\}$ $\{12,17,21\},\{17,24,27\},\{15,21,26\},\{3,8,26\},\{4,23,31\},\{8,19,28\},\{7,18,21\}$, $\{4,21,27\},\{3,11,24\},\{5,24,32\},\{12,20,30\},\{2,8,29\},\{1,14,32\},\{4,10,24\}$, $\{2,20,24\},\{12,16,24\},\{0,11,28\},\{15,18,29\},\{1,18,24\},\{6,9,31\},\{1,8,17\}$, $\{2,14,28\},\{0,20,29\},\{10,16,28\},\{0,17,26\},\{5,18,27\},\{2,21,30\},\{6,12,27\}$, $\{3,21,25\},\{13,24,28\},\{11,15,27\},\{0,16,23\},\{3,20,28\},\{14,27,30\},\{1,16,31\}$, $\{0,8,12\},\{1,23,27\},\{8,22,27\},\{1,11,19\},\{7,20,31\},\{4,25,32\},\{1,7,25\}$, $\{10,18,26\},\{5,13,20\},\{1,5,10\},\{6,17,20\},\{9,17,25\},\{1,12,15\},\{3,17,22\}$, $\{11,14,21\},\{8,13,32\},\{4,17,30\},\{4,14,22\},\{8,15,23\},\{13,25,29\},\{10,13,27\}$, $\{3,7,27\},\{2,4,15\},\{15,24,30\},\{3,14,29\},\{0,21,31\},\{14,18,25\},\{4,9,26\}$, $\{10,14,20\},\{4,7,16\},\{8,16,20\},\{5,9,28\},\{5,17,29\},\{9,21,29\},\{13,17,31\}$, $\{5,11,30\},\{5,12,19\},\{12,22,26\},\{10,23,29\},\{0,22,30\},\{7,17,28\},\{2,9,13\}$, $\{19,24,31\},\{0,4,13\},\{1,20,26\},\{8,14,31\},\{8,10,30\},\{6,13,23\},\{7,23,30\}$, $\{8,21,24\},\{11,13,18\},\{6,10,19\},\{7,14,19\},\{1,6,22\},\{1,13,21\},\{16,19,30\}$, $\{2,18,22\},\{12,29,32\},\{2,11,31\},\{16,22,29\},\{2,6,16\},\{0,19,27\},\{5,23,26\}$, $\{2,10,17\},\{6,11,29\},\{0,3,18\},\{6,14,24\},\{17,19,32\},\{9,15,19\},\{4,12,28\}$, $\{13,19,26\},\{2,26,27\},\{2,23,32\},\{4,19,29\},\{4,8,18\},\{15,31,32\},\{18,23,28\}$, $\{0,7,32\},\{11,17,23\},\{2,19,25\},\{5,7,15\},\{15,20,25\},\{1,9,30\},\{10,22,25\}$, $\{3,13,30\},\{3,6,15\},\{7,24,29\},\{9,16,27\},\{6,18,30\},\{9,14,23\},\{9,20,22\}$, $\{9,18,32\},\{11,22,32\},\{2,7,12\},\{0,5,14\},\{14,16,26\},\{7,13,22\},\{3,9,12\}$, $\{5,16,21\},\{7,11,26\},\{15,22,28\},\{5,22,31\},\{12,23,25\},\{4,11,20\},\{3,10,31\}$, $\{5,8,25\},\{20,27,32\},\{12,18,31\},\{10,21,32\},\{11,16,25\},\{6,26,32\},\{3,16,32\}$,
$\{3,19,23\},\{1,28,29\},\{0,10,15\},\{0,9,24\},\{0,6,25\},\{6,21,28\}$

## B Cases from Lemmas 22 and 24

$$
\begin{aligned}
\mathbf{B ( a )} \quad I(13,25)=[0,22] \\
I(13,25)=[0,22]
\end{aligned}
$$

Consider the following STS(13): $\{0,1,2\},\{0,3,4\},\{0,5,6\},\{0,7,8\},\{0,9,10\}$, $\{0,11,12\},\{1,3,5\},\{1,4,7\},\{1,6,8\},\{1,9,11\},\{1,10,12\},\{2,3,9\},\{2,4,5\}$, $\{2,6,10\},\{2,7,12\},\{2,8,11\},\{3,6,11\},\{3,7,10\},\{3,8,12\},\{4,6,12\},\{4,8,9\}$, $\{4,10,11\},\{5,7,11\},\{5,8,10\},\{5,9,12\},\{6,7,9\}$

The STS(25) given below intersect this system in the indicated number of blocks. intersection 22 :
$\{0,1,2\},\{0,3,4\},\{5,8,10\},\{0,7,8\},\{0,9,10\},\{0,11,12\},\{1,3,5\},\{1,4,7\}$, $\{6,7,9\},\{1,9,11\},\{1,10,12\},\{2,3,9\},\{2,4,5\},\{2,6,10\},\{2,7,12\}$,
$\{2,8,11\},\{3,6,11\},\{5,7,11\},\{3,8,12\},\{4,6,12\},\{4,8,9\},\{5,9,12\}$
$\{4,16,17\},\{5,13,19\},\{8,13,22\},\{0,20,22\},\{7,15,18\},\{10,11,17\}$, $\{1,16,22\},\{0,18,24\},\{9,13,20\},\{2,22,23\},\{2,14,21\},\{4,15,24\}$,
$\{8,14,23\},\{2,13,17\},\{4,11,18\},\{7,14,17\},\{9,19,21\},\{3,23,24\}$,
$\{11,15,20\},\{0,5,15\},\{10,21,23\},\{1,17,20\},\{10,14,15\},\{5,16,23\}$, $\{5,6,14\},\{12,17,23\},\{6,8,16\},\{7,20,23\},\{8,15,21\},\{1,18,21\},\{0,13,21\}$, $\{1,6,19\},\{9,18,23\},\{8,17,18\},\{6,22,24\},\{6,17,21\},\{4,10,13\},\{4,19,23\}$, $\{0,6,23\},\{1,8,24\},\{5,17,24\},\{11,14,24\},\{2,18,19\},\{12,16,20\}$, $\{11,16,21\},\{8,19,20\},\{2,15,16\},\{11,19,22\},\{12,21,24\},\{7,16,19\}$, $\{5,20,21\},\{10,16,18\},\{1,15,23\},\{7,13,24\},\{9,14,22\},\{10,19,24\}$, $\{12,14,19\},\{9,15,17\},\{1,13,14\},\{3,14,18\},\{6,13,15\},\{0,17,19\}$,
$\{5,18,22\},\{12,15,22\},\{4,14,20\},\{2,20,24\},\{3,10,20\},\{3,17,22\}$, $\{6,18,20\},\{7,10,22\},\{9,16,24\},\{3,15,19\},\{3,7,21\},\{0,14,16\},\{4,21,22\}$, $\{11,13,23\},\{3,13,16\},\{12,13,18\}$
intersection 21:
$\{0,1,2\},\{0,3,4\},\{0,5,6\},\{0,7,8\},\{0,9,10\},\{0,11,12\},\{5,7,11\},\{1,4,7\}$,
$\{1,6,8\},\{4,10,11\},\{5,9,12\},\{2,3,9\},\{2,4,5\},\{2,6,10\},\{6,7,9\},\{2,8,11\}$, $\{5,8,10\},\{3,7,10\},\{3,8,12\},\{4,6,12\},\{4,8,9\}$
$\{5,14,18\},\{2,18,22\},\{11,17,24\},\{0,15,20\},\{7,16,19\},\{2,14,17\}$,
$\{8,15,19\},\{2,13,16\},\{1,18,24\},\{7,13,18\},\{9,21,23\},\{6,19,23\},\{1,9,17\}$, $\{3,6,18\},\{1,19,20\},\{5,19,21\},\{5,16,17\},\{3,11,19\},\{6,15,24\},\{2,15,21\}$, $\{3,23,24\},\{8,17,18\},\{10,17,19\},\{1,11,22\},\{9,16,18\},\{2,12,23\}$,
$\{1,5,23\},\{12,14,19\},\{1,12,21\},\{1,10,14\},\{7,14,15\},\{0,18,19\}$,
$\{2,19,24\},\{6,16,21\},\{4,14,16\},\{1,13,15\},\{12,15,18\},\{0,17,21\}$,
$\{4,19,22\},\{9,13,19\},\{6,14,20\},\{3,5,15\},\{7,17,23\},\{4,18,20\},\{5,13,20\}$,
$\{6,11,13\},\{11,15,16\},\{4,15,17\},\{4,13,23\},\{16,20,23\},\{7,21,22\}$,
$\{6,17,22\},\{12,20,22\},\{11,14,21\},\{10,15,23\},\{3,17,20\},\{5,22,24\}$, $\{1,3,16\},\{10,13,22\},\{3,14,22\},\{7,12,24\},\{4,21,24\},\{0,16,24\}$, $\{10,18,21\},\{8,14,23\},\{9,14,24\},\{9,15,22\},\{9,11,20\},\{0,22,23\}$, $\{10,12,16\},\{8,13,24\},\{12,13,17\},\{8,16,22\},\{10,20,24\},\{3,13,21\}$, $\{11,18,23\},\{2,7,20\},\{8,20,21\},\{0,13,14\}$
intersection 20 :
$\{5,9,12\},\{0,3,4\},\{0,5,6\},\{0,7,8\},\{0,9,10\},\{0,11,12\},\{1,3,5\},\{1,4,7\}$, $\{1,6,8\},\{1,9,11\},\{1,10,12\},\{5,7,11\},\{2,4,5\},\{2,6,10\},\{5,8,10\}$, $\{2,8,11\},\{3,6,11\},\{3,7,10\},\{4,8,9\},\{4,10,11\}$
$\{3,14,18\},\{8,18,22\},\{0,2,16\},\{9,13,24\},\{0,15,22\},\{5,16,24\},\{3,13,23\}$, $\{8,17,20\},\{2,3,19\},\{11,19,21\},\{5,19,22\},\{4,6,24\},\{12,17,19\}$, $\{10,19,23\},\{8,16,21\},\{7,21,22\},\{5,14,15\},\{4,22,23\},\{1,17,24\}$, $\{7,13,19\},\{9,21,23\},\{3,20,21\},\{2,7,24\},\{3,22,24\},\{2,15,21\}$, $\{12,18,24\},\{6,9,15\},\{1,20,22\},\{11,16,20\},\{3,12,16\},\{1,16,18\}$, $\{11,13,18\},\{10,21,24\},\{1,13,15\},\{1,14,21\},\{5,13,21\},\{0,18,23\}$, $\{4,15,19\},\{3,8,15\},\{15,20,24\},\{6,7,20\},\{7,12,14\},\{11,14,22\}$, $\{11,15,17\},\{6,14,17\},\{9,19,20\},\{8,14,23\},\{0,14,24\},\{0,1,19\}$, $\{2,12,22\},\{7,9,18\},\{7,17,23\},\{14,16,19\},\{11,23,24\},\{6,18,19\}$, $\{4,16,17\},\{5,20,23\},\{1,2,23\},\{7,15,16\},\{5,17,18\},\{9,16,22\},\{2,13,17\}$, $\{3,9,17\},\{6,16,23\},\{6,12,21\},\{10,14,20\},\{10,15,18\},\{2,18,20\}$, $\{6,13,22\},\{4,12,20\},\{8,19,24\},\{2,9,14\},\{4,18,21\},\{10,13,16\}$, $\{0,13,20\},\{12,15,23\},\{0,17,21\},\{10,17,22\},\{4,13,14\},\{8,12,13\}$ intersection 19:
$\{0,1,2\},\{5,9,12\},\{0,5,6\},\{4,10,11\},\{6,7,9\},\{0,11,12\},\{1,3,5\}$, $\{4,6,12\},\{4,8,9\},\{1,9,11\},\{1,10,12\},\{2,3,9\},\{5,7,11\},\{2,6,10\}$, $\{2,7,12\},\{2,8,11\},\{5,8,10\},\{3,7,10\},\{3,8,12\}$
$\{2,14,21\},\{4,5,16\},\{5,21,24\},\{12,19,21\},\{0,3,18\},\{5,19,23\},\{2,20,23\}$, $\{9,10,13\},\{7,17,23\},\{11,14,17\},\{3,13,17\},\{13,21,22\},\{6,16,17\}$, $\{5,18,22\},\{7,13,24\},\{8,14,16\},\{9,21,23\},\{6,8,23\},\{3,4,21\},\{3,11,15\}$, $\{11,16,21\},\{1,15,20\},\{8,20,22\},\{13,14,19\},\{6,15,24\},\{11,18,20\}$, $\{10,17,24\},\{3,23,24\},\{0,15,22\},\{9,15,17\},\{4,17,18\},\{4,7,14\}$,
$\{3,16,19\},\{4,20,24\},\{3,6,20\},\{0,7,20\},\{1,14,23\},\{6,14,18\},\{17,20,21\}$, $\{0,8,17\},\{8,15,21\},\{0,13,23\},\{0,16,24\},\{1,7,22\},\{10,14,15\}$,
$\{12,14,24\},\{3,14,22\},\{7,15,16\},\{2,4,22\},\{12,16,23\},\{0,4,19\}$,
$\{2,13,16\},\{11,19,24\},\{1,8,24\},\{4,15,23\},\{12,15,18\},\{6,11,13\}$,
$\{1,4,13\},\{1,16,18\},\{8,13,18\},\{2,18,24\},\{5,14,20\},\{9,22,24\},\{0,9,14\}$, $\{1,6,21\},\{10,18,23\},\{0,10,21\},\{6,19,22\},\{11,22,23\},\{2,15,19\}$,
$\{7,18,21\},\{9,16,20\},\{7,8,19\},\{5,13,15\},\{9,18,19\},\{2,5,17\},\{10,19,20\}$, $\{10,16,22\},\{1,17,19\},\{12,17,22\},\{12,13,20\}$
intersection 18:
$\{0,1,2\},\{5,7,11\},\{0,5,6\},\{4,6,12\},\{0,9,10\},\{3,8,12\},\{5,8,10\},\{1,4,7\}$, $\{4,8,9\},\{6,7,9\},\{1,10,12\},\{2,3,9\},\{2,4,5\},\{2,6,10\},\{5,9,12\},\{2,8,11\}$, $\{3,6,11\},\{3,7,10\}$
$\{4,18,19\},\{3,5,22\},\{0,14,24\},\{3,23,24\},\{3,14,17\},\{1,3,21\},\{9,19,21\}$, $\{1,17,20\},\{6,13,15\},\{7,12,13\},\{8,17,24\},\{9,20,22\},\{3,4,13\},\{1,13,19\}$, $\{9,15,23\},\{4,16,23\},\{8,19,23\},\{2,13,16\},\{7,18,22\},\{9,11,24\},\{0,4,22\}$, $\{6,19,20\},\{9,13,17\},\{5,13,24\},\{6,17,18\},\{10,13,18\},\{3,15,19\}$, $\{11,14,16\},\{12,22,23\},\{4,10,24\},\{0,7,15\},\{4,11,20\},\{5,15,18\}$, $\{0,11,19\},\{12,20,24\},\{0,13,23\},\{2,21,22\},\{4,14,21\},\{2,15,20\}$, $\{1,11,18\},\{13,14,20\},\{1,8,15\},\{1,22,24\},\{10,11,23\},\{2,17,23\}$, $\{10,14,19\},\{18,21,23\},\{7,19,24\},\{11,15,22\},\{7,8,21\},\{7,16,17\}$, $\{15,16,24\},\{0,3,16\},\{10,17,22\},\{1,5,16\},\{0,8,20\},\{12,14,15\}$, $\{16,19,22\},\{5,17,19\},\{2,7,14\},\{2,18,24\},\{1,9,14\},\{10,16,20\}$, $\{0,17,21\},\{12,16,21\},\{0,12,18\},\{1,6,23\},\{5,20,21\},\{8,14,18\}$, $\{11,12,17\},\{8,13,22\},\{3,18,20\},\{10,15,21\},\{2,12,19\},\{6,14,22\}$, $\{4,15,17\},\{9,16,18\},\{7,20,23\},\{5,14,23\},\{6,21,24\},\{6,8,16\},\{11,13,21\}$ intersection 17:
$\{0,1,2\},\{0,3,4\},\{0,5,6\},\{4,10,11\},\{0,9,10\},\{0,11,12\},\{4,8,9\}$, $\{5,8,10\},\{5,9,12\},\{1,9,11\},\{6,7,9\},\{3,7,10\},\{2,4,5\},\{2,6,10\}$, $\{4,6,12\},\{2,8,11\},\{3,6,11\}$
$\{9,16,18\},\{6,18,19\},\{3,9,19\},\{10,21,24\},\{4,17,24\},\{8,19,20\}$,
$\{7,12,13\},\{0,17,19\},\{0,14,20\},\{13,16,20\},\{4,20,22\},\{11,13,24\}$, $\{11,14,18\},\{6,22,24\},\{3,8,21\},\{4,7,15\},\{12,19,24\},\{12,18,22\}$, $\{1,12,20\},\{5,13,22\},\{2,7,22\},\{3,13,18\},\{9,13,15\},\{2,3,20\},\{7,8,23\}$, $\{8,17,22\},\{0,21,22\},\{1,19,21\},\{0,13,23\},\{8,15,18\},\{2,15,19\}$, $\{13,17,21\},\{2,13,14\},\{2,12,16\},\{4,18,21\},\{0,7,18\},\{7,20,24\}$, $\{9,22,23\},\{5,15,17\},\{0,8,16\},\{3,16,24\},\{7,14,19\},\{1,16,17\},\{7,16,21\}$, $\{0,15,24\},\{3,5,14\},\{1,4,13\},\{4,19,23\},\{3,12,17\},\{10,13,19\},\{2,21,23\}$, $\{1,5,7\},\{12,15,21\},\{1,6,14\},\{1,8,24\},\{11,15,20\},\{4,14,16\},\{10,12,23\}$, $\{6,8,13\},\{5,11,21\},\{5,16,19\},\{3,15,23\},\{6,20,21\},\{6,17,23\},\{2,9,24\}$,
$\{7,11,17\},\{14,15,22\},\{1,10,15\},\{5,20,23\},\{10,14,17\},\{8,12,14\}$,
$\{6,15,16\},\{9,14,21\},\{10,16,22\},\{11,16,23\},\{9,17,20\},\{2,17,18\}$,
$\{5,18,24\},\{11,19,22\},\{14,23,24\},\{10,18,20\},\{1,18,23\},\{1,3,22\}$
intersection 16 :
$\{0,1,2\},\{3,7,10\},\{4,6,12\},\{0,7,8\},\{0,9,10\},\{0,11,12\},\{1,3,5\},\{5,8,10\}$, $\{1,6,8\},\{3,8,12\},\{1,10,12\},\{2,3,9\},\{2,4,5\},\{2,6,10\},\{4,8,9\},\{5,7,11\}$
$\{6,16,17\},\{7,12,20\},\{2,22,23\},\{10,20,24\},\{0,6,19\},\{10,13,19\}$, $\{6,14,24\},\{5,15,24\},\{2,7,24\},\{7,16,19\},\{3,14,19\},\{7,13,23\},\{2,11,15\}$, $\{4,18,21\},\{10,11,22\},\{1,14,23\},\{2,18,19\},\{12,14,15\},\{0,13,17\}$, $\{6,15,21\},\{11,14,16\},\{0,18,24\},\{19,20,23\},\{10,21,23\},\{1,21,24\}$, $\{6,13,22\},\{12,17,18\},\{6,7,18\},\{0,4,20\},\{15,20,22\},\{5,13,21\}$, $\{8,17,22\},\{4,7,15\},\{9,15,18\},\{1,9,19\},\{5,6,20\},\{4,13,14\},\{2,12,13\}$, $\{5,16,18\},\{4,10,17\},\{5,9,23\},\{4,19,22\},\{14,20,21\},\{10,14,18\}$, $\{8,13,24\},\{11,13,18\},\{5,12,22\},\{0,3,15\},\{1,18,20\},\{5,17,19\}$, $\{12,19,21\},\{1,7,22\},\{7,9,21\},\{3,6,23\},\{3,4,24\},\{8,11,20\},\{8,18,23\}$, $\{2,16,20\},\{7,14,17\},\{2,17,21\},\{3,18,22\},\{2,8,14\},\{0,21,22\}$, $\{10,15,16\},\{12,23,24\},\{8,16,21\},\{16,22,24\},\{3,13,16\},\{3,11,21\}$,
$\{9,13,20\},\{9,17,24\},\{1,11,17\},\{9,14,22\},\{6,9,11\},\{8,15,19\},\{4,11,23\}$, $\{1,4,16\},\{0,16,23\},\{11,19,24\},\{1,13,15\},\{0,5,14\},\{9,12,16\}$, $\{15,17,23\},\{3,17,20\}$
intersection 15 :
$\{0,1,2\},\{2,8,11\},\{3,8,12\},\{0,7,8\},\{0,9,10\},\{4,8,9\},\{1,3,5\},\{1,4,7\}$, $\{1,6,8\},\{4,6,12\},\{1,10,12\},\{6,7,9\},\{3,7,10\},\{2,6,10\},\{2,7,12\}$
$\{5,10,24\},\{5,8,20\},\{0,12,23\},\{14,23,24\},\{0,5,13\},\{11,19,24\}$, $\{10,11,22\},\{1,11,21\},\{12,18,22\},\{13,15,23\},\{9,12,24\},\{3,13,18\}$, $\{8,19,23\},\{2,16,21\},\{9,11,23\},\{0,17,18\},\{0,20,21\},\{6,13,20\}$, $\{13,21,24\},\{5,9,14\},\{6,11,15\},\{4,11,20\},\{16,20,22\},\{7,18,20\}$,
$\{2,4,15\},\{11,12,13\},\{7,11,17\},\{10,15,16\},\{0,11,16\},\{16,18,23\}$,
$\{6,22,23\},\{0,15,22\},\{7,14,21\},\{4,16,19\},\{14,18,19\},\{2,5,19\},\{4,5,17\}$,
$\{8,10,14\},\{3,15,20\},\{9,19,20\},\{10,17,21\},\{8,13,22\},\{2,20,24\}$,
$\{9,17,22\},\{3,17,24\},\{4,13,14\},\{0,6,14\},\{0,4,24\},\{0,3,19\},\{6,17,19\}$, $\{8,15,24\},\{12,19,21\},\{9,15,21\},\{1,16,24\},\{12,17,20\},\{5,11,18\}$, $\{2,9,18\},\{1,15,18\},\{5,7,23\},\{3,6,21\},\{4,21,23\},\{4,10,18\},\{5,6,16\}$, $\{3,4,22\},\{14,15,17\},\{7,15,19\},\{1,17,23\},\{2,14,22\},\{7,22,24\}$, $\{1,19,22\},\{5,21,22\},\{5,12,15\},\{3,11,14\},\{1,14,20\},\{6,18,24\}$, $\{8,16,17\},\{10,20,23\},\{12,14,16\},\{10,13,19\},\{1,9,13\},\{7,13,16\}$, $\{8,18,21\},\{3,9,16\},\{2,3,23\},\{2,13,17\}$
intersection 14:
$\{5,7,11\},\{4,8,9\},\{3,8,12\},\{5,9,12\},\{0,9,10\},\{4,10,11\},\{2,8,11\}$, $\{1,4,7\},\{6,7,9\},\{1,9,11\},\{1,10,12\},\{2,3,9\},\{2,7,12\},\{2,6,10\}$
$\{11,16,19\},\{2,19,21\},\{9,13,19\},\{10,15,21\},\{12,13,17\},\{5,15,16\}$, $\{8,14,20\},\{7,8,21\},\{2,18,22\},\{9,18,20\},\{3,15,17\},\{3,10,23\},\{8,10,16\}$, $\{11,14,17\},\{9,14,22\},\{9,15,24\},\{3,5,19\},\{4,18,19\},\{14,21,24\}$, $\{3,13,20\},\{4,23,24\},\{11,12,18\},\{7,10,13\},\{5,20,23\},\{1,15,18\}$, $\{15,19,20\},\{4,17,21\},\{2,4,20\},\{6,12,15\},\{0,17,22\},\{10,17,18\}$, $\{0,4,12\},\{4,15,22\},\{9,21,23\},\{1,2,14\},\{6,11,20\},\{1,6,19\},\{0,13,14\}$, $\{7,14,15\},\{7,16,24\},\{0,11,24\},\{5,8,24\},\{13,18,24\},\{5,18,21\},\{0,1,23\}$, $\{7,22,23\},\{0,3,21\},\{2,13,15\},\{1,5,17\},\{11,15,23\},\{4,6,16\},\{3,7,18\}$, $\{8,18,23\},\{2,17,24\},\{3,6,24\},\{6,8,17\},\{12,16,21\},\{0,7,19\},\{4,5,13\}$, $\{3,11,22\},\{12,19,24\},\{10,20,24\},\{0,8,15\},\{12,14,23\},\{2,16,23\}$, $\{11,13,21\},\{8,19,22\},\{1,20,21\},\{1,8,13\},\{1,3,16\},\{13,16,22\}$, $\{5,10,22\},\{12,20,22\},\{17,19,23\},\{6,21,22\},\{10,14,19\},\{0,2,5\}$, $\{0,16,20\},\{9,16,17\},\{7,17,20\},\{5,6,14\},\{3,4,14\},\{6,13,23\},\{1,22,24\}$, $\{14,16,18\},\{0,6,18\}$
intersection 13 :
$\{6,7,9\},\{2,8,11\},\{0,5,6\},\{0,7,8\},\{3,6,11\},\{0,11,12\},\{1,3,5\},\{1,4,7\}$, $\{2,7,12\},\{4,6,12\},\{5,8,10\},\{2,3,9\},\{3,7,10\}$
$\{15,21,24\},\{1,17,18\},\{1,10,19\},\{5,14,24\},\{2,13,17\},\{8,16,20\}$,
$\{2,18,22\},\{7,11,23\},\{7,22,24\},\{0,13,16\},\{7,14,18\},\{8,13,23\},\{1,8,24\}$, $\{2,4,15\},\{5,20,23\},\{8,18,19\},\{8,9,22\},\{3,16,18\},\{9,16,17\},\{4,5,16\}$,
$\{12,19,20\},\{11,21,22\},\{9,10,14\},\{4,10,24\},\{11,20,24\},\{8,12,14\}$, $\{5,9,18\},\{1,14,15\},\{10,11,18\},\{4,20,22\},\{1,11,13\},\{6,16,24\}$, $\{4,13,14\},\{14,16,22\},\{1,12,16\},\{3,13,24\},\{0,17,23\},\{7,16,19\}$, $\{14,17,20\},\{9,15,19\},\{0,9,21\},\{3,8,15\},\{5,12,21\},\{3,19,23\}$, $\{10,12,17\},\{6,19,22\},\{15,17,22\},\{0,18,20\},\{3,20,21\},\{2,5,19\}$, $\{5,11,17\},\{11,14,19\},\{7,17,21\},\{4,8,21\},\{10,22,23\},\{13,19,21\}$, $\{6,8,17\},\{0,3,14\},\{6,18,21\},\{5,13,22\},\{14,21,23\},\{9,23,24\},\{5,7,15\}$, $\{7,13,20\},\{9,12,13\},\{2,16,23\},\{4,9,11\},\{0,4,19\},\{12,18,24\},\{1,2,21\}$, $\{10,16,21\},\{0,2,24\},\{12,15,23\},\{2,10,20\},\{2,6,14\},\{1,9,20\},\{6,10,13\}$, $\{3,4,17\},\{1,6,23\},\{0,1,22\},\{3,12,22\},\{0,10,15\},\{11,15,16\},\{17,19,24\}$, $\{13,15,18\},\{6,15,20\},\{4,18,23\}$
intersection 12 :
$\{5,8,10\},\{0,3,4\},\{2,7,12\},\{4,6,12\},\{2,8,11\},\{0,11,12\},\{1,3,5\}$, $\{2,4,5\},\{4,8,9\},\{3,6,11\},\{4,10,11\},\{5,7,11\}$
$\{1,9,18\},\{9,11,13\},\{8,14,15\},\{5,12,24\},\{3,16,19\},\{5,6,20\},\{0,2,18\}$, $\{2,9,22\},\{2,6,14\},\{3,12,14\},\{4,14,22\},\{8,13,21\},\{13,20,23\},\{1,13,24\}$, $\{0,16,22\},\{10,13,22\},\{5,18,23\},\{12,13,16\},\{1,7,14\},\{5,9,16\}$, $\{1,15,19\},\{14,17,19\},\{4,15,17\},\{2,16,23\},\{3,21,22\},\{14,18,21\}$, $\{3,7,13\},\{18,19,20\},\{6,13,15\},\{6,9,24\},\{7,18,22\},\{5,13,14\},\{7,10,21\}$, $\{3,17,23\},\{17,18,24\},\{1,6,22\},\{5,17,22\},\{1,12,17\},\{4,7,20\}$,
$\{10,12,19\},\{6,7,19\},\{11,17,21\},\{8,19,22\},\{2,17,20\},\{12,20,22\}$, $\{1,2,10\},\{0,6,10\},\{11,15,18\},\{7,9,17\},\{1,11,20\},\{10,15,20\}$,
$\{12,15,23\},\{1,16,21\},\{0,5,19\},\{0,9,15\},\{7,15,16\},\{0,1,8\},\{15,22,24\}$,
$\{11,22,23\},\{9,19,23\},\{3,9,20\},\{6,21,23\},\{7,8,23\},\{1,4,23\},\{11,14,16\}$, $\{14,20,24\},\{8,12,18\},\{4,13,18\},\{8,16,20\},\{6,16,18\},\{11,19,24\}$,
$\{0,13,17\},\{2,13,19\},\{5,15,21\},\{4,16,24\},\{10,23,24\},\{10,16,17\}$,
$\{3,8,24\},\{3,10,18\},\{2,21,24\},\{9,12,21\},\{9,10,14\},\{6,8,17\},\{2,3,15\}$,
$\{0,7,24\},\{4,19,21\},\{0,14,23\},\{0,20,21\}$
intersection 11:
$\{4,8,9\},\{6,7,9\},\{0,5,6\},\{4,6,12\},\{0,9,10\},\{5,7,11\},\{3,8,12\},\{2,3,9\}$, $\{2,4,5\},\{1,9,11\},\{1,10,12\}$
$\{4,17,18\},\{7,8,22\},\{11,16,21\},\{8,10,14\},\{8,18,21\},\{8,11,17\}$, $\{0,11,14\},\{5,12,19\},\{16,17,20\},\{12,13,16\},\{12,21,23\},\{5,13,22\}$,
$\{3,13,20\},\{9,12,15\},\{0,23,24\},\{7,10,18\},\{0,8,19\},\{3,5,10\},\{6,10,11\}$,
$\{0,12,18\},\{3,6,23\},\{4,15,22\},\{6,16,22\},\{3,14,19\},\{9,20,22\},\{4,10,13\}$, $\{9,19,21\},\{1,3,24\},\{5,17,21\},\{1,8,13\},\{10,15,16\},\{0,2,16\},\{9,18,24\}$, $\{6,13,19\},\{5,9,23\},\{10,17,24\},\{7,14,15\},\{1,22,23\},\{7,16,19\}$, $\{3,21,22\},\{11,12,22\},\{1,6,18\},\{6,8,15\},\{2,13,18\},\{2,10,22\},\{9,13,17\}$, $\{12,14,17\},\{18,20,23\},\{4,14,23\},\{1,2,14\},\{9,14,16\},\{1,5,15\}$,
$\{11,15,18\},\{1,17,19\},\{6,14,20\},\{0,7,13\},\{5,8,20\},\{13,14,21\}$,
$\{11,13,23\},\{13,15,24\},\{3,16,18\},\{3,7,17\},\{1,4,16\},\{1,7,20\},\{4,19,24\}$,
$\{2,7,23\},\{18,19,22\},\{7,12,24\},\{10,20,21\},\{5,16,24\},\{5,14,18\}$,
$\{0,3,15\},\{2,6,17\},\{10,19,23\},\{8,16,23\},\{0,17,22\},\{11,20,24\}$,
$\{15,19,20\},\{2,15,21\},\{15,17,23\},\{2,12,20\},\{3,4,11\},\{14,22,24\}$,
$\{2,8,24\},\{2,11,19\},\{0,1,21\},\{6,21,24\},\{4,7,21\},\{0,4,20\}$
intersection 10 :
$\{6,7,9\},\{5,9,12\},\{5,8,10\},\{4,6,12\},\{0,9,10\},\{0,11,12\},\{2,7,12\}$, $\{2,4,5\},\{3,8,12\},\{2,6,10\}$
$\{11,16,21\},\{0,21,23\},\{2,9,19\},\{1,10,13\},\{9,17,22\},\{10,18,21\}$, $\{1,12,20\},\{12,13,23\},\{8,18,19\},\{5,7,24\},\{0,1,7\},\{5,13,17\},\{16,18,20\}$, $\{9,13,14\},\{17,23,24\},\{1,8,14\},\{4,14,24\},\{12,16,17\},\{3,13,22\}$,
$\{9,15,23\},\{4,18,22\},\{7,11,19\},\{12,14,18\},\{7,15,22\},\{5,6,23\},\{1,2,22\}$, $\{7,17,20\},\{3,9,21\},\{11,13,24\},\{1,15,21\},\{6,16,22\},\{0,5,15\},\{2,18,23\}$, $\{9,20,24\},\{1,6,11\},\{15,16,24\},\{8,13,20\},\{1,4,9\},\{4,8,15\},\{10,17,19\}$, $\{6,14,17\},\{3,10,24\},\{5,20,21\},\{0,18,24\},\{2,8,24\},\{10,20,22\}$,
$\{12,19,21\},\{6,21,24\},\{7,8,23\},\{8,21,22\},\{0,3,16\},\{0,13,19\},\{5,11,22\}$, $\{12,22,24\},\{1,16,23\},\{1,19,24\},\{8,11,17\},\{1,5,18\},\{15,17,18\}$,
$\{3,11,23\},\{3,6,18\},\{14,15,19\},\{19,22,23\},\{14,20,23\},\{0,2,20\}$,
$\{5,14,16\},\{4,11,20\},\{9,11,18\},\{7,13,18\},\{0,4,17\},\{3,5,19\},\{7,10,16\}$,
$\{6,13,15\},\{0,6,8\},\{2,17,21\},\{2,3,14\},\{4,10,23\},\{0,14,22\},\{10,12,15\}$,
$\{6,19,20\},\{3,15,20\},\{10,11,14\},\{2,11,15\},\{7,14,21\},\{3,4,7\},\{1,3,17\}$,
$\{4,13,21\},\{2,13,16\},\{8,9,16\},\{4,16,19\}$
intersection 9:
$\{0,1,2\},\{5,9,12\},\{2,4,5\},\{2,3,9\},\{0,9,10\},\{4,10,11\},\{1,9,11\},\{1,4,7\}$, $\{1,6,8\}$
$\{12,15,23\},\{10,13,16\},\{7,17,18\},\{15,16,21\},\{5,11,14\},\{12,18,22\}$,
$\{1,19,23\},\{3,18,24\},\{1,5,17\},\{6,11,15\},\{11,17,23\},\{7,8,16\},\{6,9,16\}$, $\{0,16,17\},\{10,12,19\},\{0,3,7\},\{3,4,22\},\{0,5,8\},\{16,19,20\},\{2,14,23\}$, $\{3,6,13\},\{14,19,21\},\{8,11,24\},\{9,18,21\},\{9,13,23\},\{5,10,21\},\{3,8,21\}$, $\{9,17,22\},\{5,20,23\},\{17,19,24\},\{3,15,17\},\{12,13,24\},\{2,17,20\}$,
$\{12,14,16\},\{22,23,24\},\{2,11,21\},\{12,17,21\},\{5,13,15\},\{4,16,23\}$,
$\{4,13,17\},\{0,13,21\},\{6,12,20\},\{8,13,20\},\{2,13,18\},\{2,16,24\}$,
$\{8,19,22\},\{3,14,20\},\{8,10,17\},\{4,14,18\},\{7,15,19\},\{5,6,18\},\{2,6,19\}$,
$\{4,9,19\},\{6,10,24\},\{3,10,23\},\{1,3,12\},\{5,7,24\},\{10,15,18\},\{20,21,22\}$, $\{10,14,22\},\{0,18,19\},\{9,15,24\},\{8,9,14\},\{6,7,22\},\{1,10,20\},\{0,4,12\}$, $\{11,18,20\},\{4,20,24\},\{7,9,20\},\{5,16,22\},\{2,8,12\},\{3,5,19\},\{7,11,12\}$, $\{0,11,22\},\{8,18,23\},\{2,15,22\},\{0,14,24\},\{2,7,10\},\{3,11,16\},\{0,15,20\}$, $\{4,6,21\},\{7,21,23\},\{11,13,19\},\{0,6,23\},\{1,21,24\},\{4,8,15\},\{1,16,18\}$, $\{1,13,22\},\{1,14,15\},\{7,13,14\},\{6,14,17\}$
intersection 8:
$\{5,9,12\},\{0,3,4\},\{4,6,12\},\{2,7,12\},\{1,10,12\},\{0,11,12\},\{2,8,11\}$, $\{1,4,7\}$
$\{2,3,21\},\{5,17,23\},\{0,9,17\},\{0,2,16\},\{3,10,17\},\{0,10,22\},\{1,16,21\}$, $\{6,9,14\},\{8,13,17\},\{3,5,6\},\{2,9,20\},\{1,17,24\},\{8,9,23\},\{6,18,20\}$, $\{6,10,13\},\{1,3,8\},\{7,9,13\},\{7,17,22\},\{10,15,16\},\{4,11,14\},\{9,11,21\}$, $\{14,16,24\},\{13,15,20\},\{2,19,22\},\{7,11,23\},\{4,17,20\},\{1,11,13\}$,
$\{14,15,17\},\{3,13,19\},\{6,19,24\},\{7,10,24\},\{6,7,15\},\{5,10,21\},\{0,7,21\}$,
$\{4,22,24\},\{9,18,24\},\{10,18,19\},\{11,15,22\},\{7,16,18\},\{8,12,22\}$, $\{15,21,23\},\{2,13,24\},\{1,2,14\},\{8,16,20\},\{20,23,24\},\{14,18,21\}$, $\{3,16,23\},\{5,8,18\},\{3,9,15\},\{12,15,24\},\{16,17,19\},\{0,1,20\},\{3,11,24\}$, $\{0,5,24\},\{8,10,14\},\{2,5,15\},\{12,13,16\},\{4,8,15\},\{1,6,23\},\{4,9,10\}$, $\{5,7,20\},\{2,6,17\},\{1,15,18\},\{1,9,19\},\{2,4,18\},\{12,14,19\},\{2,10,23\}$, $\{6,21,22\},\{10,11,20\},\{7,8,19\},\{19,20,21\},\{13,22,23\},\{3,12,20\}$, $\{9,16,22\},\{0,6,8\},\{5,11,19\},\{0,13,18\},\{12,17,21\},\{0,15,19\},\{8,21,24\}$, $\{6,11,16\},\{0,14,23\},\{14,20,22\},\{12,18,23\},\{5,13,14\},\{4,19,23\}$, $\{11,17,18\},\{4,5,16\},\{1,5,22\},\{4,13,21\},\{3,7,14\},\{3,18,22\}$
intersection 7 :
$\{4,6,12\},\{1,6,8\},\{6,7,9\},\{0,7,8\},\{1,9,11\},\{5,8,10\},\{2,6,10\}$
$\{9,19,23\},\{16,20,24\},\{8,12,22\},\{6,18,20\},\{4,8,16\},\{2,8,20\},\{4,9,22\}$, $\{12,15,17\},\{10,18,19\},\{5,6,17\},\{7,13,17\},\{9,14,24\},\{0,13,18\}$, $\{4,13,20\},\{2,19,22\},\{0,23,24\},\{5,7,15\},\{3,7,22\},\{2,3,17\},\{5,20,23\}$, $\{5,11,24\},\{2,15,21\},\{0,3,9\},\{1,10,21\},\{11,16,17\},\{0,2,16\},\{7,14,20\}$, $\{17,22,24\},\{8,21,23\},\{9,10,20\},\{10,13,14\},\{1,3,14\},\{3,5,12\},\{3,10,16\}$, $\{7,16,21\},\{2,9,12\},\{3,8,19\},\{0,17,20\},\{0,12,14\},\{4,14,18\},\{1,2,4\}$, $\{2,5,18\},\{7,11,12\},\{3,20,21\},\{6,14,22\},\{12,16,19\},\{6,16,23\},\{5,9,16\}$, $\{2,7,24\},\{11,19,20\},\{3,6,13\},\{1,15,24\},\{7,18,23\},\{1,13,16\},\{6,21,24\}$, $\{0,6,11\},\{11,21,22\},\{9,15,18\},\{6,15,19\},\{8,11,14\},\{14,15,16\}$,
$\{4,7,10\},\{2,11,13\},\{9,13,21\},\{10,17,23\},\{3,11,18\},\{8,9,17\},\{0,1,5\}$, $\{8,18,24\},\{4,5,21\},\{3,4,24\},\{3,15,23\},\{5,13,22\},\{0,4,15\},\{8,13,15\}$, $\{1,22,23\},\{1,17,18\},\{16,18,22\},\{0,10,22\},\{10,11,15\},\{13,19,24\}$, $\{5,14,19\},\{2,14,23\},\{10,12,24\},\{15,20,22\},\{4,17,19\},\{12,13,23\}$, $\{0,19,21\},\{4,11,23\},\{14,17,21\},\{1,7,19\},\{12,18,21\},\{1,12,20\}$ intersection 6 :
$\{4,10,11\},\{2,8,11\},\{1,6,8\},\{3,6,11\},\{4,8,9\},\{0,11,12\}$
$\{17,20,22\},\{8,15,22\},\{11,14,22\},\{1,11,15\},\{0,16,21\},\{2,14,16\}$, $\{6,13,19\},\{6,17,23\},\{5,6,7\},\{10,15,20\},\{4,19,20\},\{7,11,24\},\{7,19,21\}$, $\{0,6,20\},\{11,19,23\},\{3,4,21\},\{1,19,22\},\{6,9,10\},\{10,16,19\},\{16,18,24\}$, $\{5,10,23\},\{1,12,18\},\{3,5,15\},\{3,10,14\},\{4,12,22\},\{18,21,22\}$,
$\{2,10,24\},\{6,12,21\},\{4,7,17\},\{9,11,20\},\{2,6,18\},\{12,15,23\},\{1,5,24\}$, $\{5,12,14\},\{14,18,23\},\{11,16,17\},\{4,5,13\},\{2,13,21\},\{7,13,15\}$,
$\{3,16,22\},\{1,4,16\},\{1,13,17\},\{20,21,23\},\{2,4,15\},\{0,3,24\},\{2,9,12\}$, $\{8,12,17\},\{2,5,22\},\{9,13,22\},\{10,17,18\},\{7,10,22\},\{1,9,23\},\{1,14,21\}$, $\{0,7,9\},\{8,16,20\},\{5,18,20\},\{10,12,13\},\{3,12,19\},\{2,3,17\},\{0,2,19\}$, $\{5,9,16\},\{13,16,23\},\{3,8,23\},\{7,12,16\},\{9,17,21\},\{1,2,20\},\{12,20,24\}$, $\{8,14,19\},\{6,15,16\},\{2,7,23\},\{9,14,15\},\{4,23,24\},\{6,22,24\},\{0,1,10\}$, $\{0,13,14\},\{15,21,24\},\{8,13,24\},\{3,13,20\},\{8,10,21\},\{9,19,24\}$,
$\{0,22,23\},\{5,11,21\},\{14,17,24\},\{4,6,14\},\{1,3,7\},\{0,15,17\},\{0,5,8\}$, $\{7,14,20\},\{5,17,19\},\{11,13,18\},\{15,18,19\},\{0,4,18\},\{7,8,18\},\{3,9,18\}$ intersection 5 :
$\{4,8,9\},\{2,4,5\},\{5,9,12\},\{3,7,10\},\{2,8,11\}$
$\{9,10,11\},\{12,21,22\},\{0,11,17\},\{4,15,17\},\{8,17,20\},\{6,11,23\}$, $\{1,12,23\},\{3,19,20\},\{0,1,7\},\{11,13,15\},\{1,8,14\},\{8,13,19\},\{0,4,22\}$, $\{4,14,20\},\{6,20,24\},\{4,6,21\},\{1,6,22\},\{7,8,24\},\{7,20,22\},\{5,8,21\}$, $\{3,5,24\},\{1,9,17\},\{5,15,18\},\{11,16,20\},\{9,14,23\},\{4,7,23\},\{9,15,16\}$, $\{9,18,20\},\{0,20,21\},\{11,22,24\},\{1,5,11\},\{14,16,19\},\{2,10,16\}$, $\{7,13,21\},\{3,11,21\},\{1,4,16\},\{18,19,21\},\{6,7,12\},\{1,3,18\},\{9,13,22\}$, $\{4,10,19\},\{15,20,23\},\{5,7,17\},\{11,12,14\},\{0,2,12\},\{0,8,16\}$,
$\{17,23,24\},\{2,14,18\},\{5,10,20\},\{7,16,18\},\{4,11,18\},\{0,10,23\}$,
$\{3,13,16\},\{6,8,15\},\{7,14,15\},\{2,19,22\},\{2,7,9\},\{0,15,19\},\{13,17,18\}$, $\{4,13,24\},\{0,14,24\},\{3,15,22\},\{16,21,23\},\{2,17,21\},\{18,22,23\}$, $\{1,15,21\},\{12,17,19\},\{16,17,22\},\{0,6,18\},\{1,10,13\},\{10,12,15\}$, $\{3,14,17\},\{12,16,24\},\{6,10,17\},\{8,12,18\},\{2,15,24\},\{3,4,12\}$, $\{12,13,20\},\{5,6,16\},\{1,2,20\},\{3,8,23\},\{6,13,14\},\{10,14,21\},\{2,13,23\}$, $\{10,18,24\},\{8,10,22\},\{5,14,22\},\{0,5,13\},\{7,11,19\},\{1,19,24\},\{2,3,6\}$, $\{6,9,19\},\{5,19,23\},\{9,21,24\},\{0,3,9\}$
intersection 4 :
$\{1,6,8\},\{5,7,11\},\{2,7,12\},\{0,7,8\}$
$\{12,16,17\},\{1,5,10\},\{6,10,21\},\{1,7,9\},\{14,16,20\},\{5,16,23\},\{4,8,17\}$, $\{1,2,3\},\{2,16,21\},\{8,14,21\},\{4,6,7\},\{1,20,21\},\{10,17,22\},\{4,9,15\}$, $\{1,12,22\},\{1,14,18\},\{3,9,11\},\{8,11,15\},\{8,18,23\},\{2,11,22\}$, $\{11,19,21\},\{0,5,21\},\{15,17,21\},\{5,6,14\},\{5,15,20\},\{0,12,15\}$, $\{10,13,23\},\{7,14,17\},\{7,20,23\},\{0,3,10\},\{4,11,13\},\{9,17,18\}$, $\{10,18,19\},\{8,12,13\},\{3,7,16\},\{2,13,14\},\{2,9,23\},\{0,9,22\},\{3,15,19\}$, $\{14,15,22\},\{6,9,16\},\{2,6,20\},\{4,21,24\},\{6,11,12\},\{6,15,23\},\{3,6,17\}$, $\{1,11,16\},\{2,8,10\},\{3,8,20\},\{4,18,20\},\{1,17,24\},\{0,1,4\},\{0,20,24\}$,
$\{13,17,20\},\{3,4,14\},\{3,5,13\},\{0,2,17\},\{12,14,19\},\{2,5,18\},\{3,21,22\}$, $\{4,22,23\},\{15,16,18\},\{5,17,19\},\{6,18,22\},\{4,5,12\},\{7,22,24\}$,
$\{9,13,21\},\{8,16,24\},\{6,19,24\},\{11,17,23\},\{9,10,14\},\{19,20,22\}$,
$\{5,9,24\},\{11,14,24\},\{13,18,24\},\{7,10,15\},\{3,12,18\},\{0,11,18\}$,
$\{1,19,23\},\{12,21,23\},\{0,14,23\},\{3,23,24\},\{0,16,19\},\{5,8,22\}$,
$\{10,12,24\},\{2,4,19\},\{10,11,20\},\{9,12,20\},\{2,15,24\},\{1,13,15\}$, $\{7,13,19\},\{7,18,21\},\{8,9,19\},\{4,10,16\},\{13,16,22\},\{0,6,13\}$
intersection 3 :
$\{5,8,10\},\{5,9,12\},\{3,7,10\}$
$\{14,17,19\},\{3,12,18\},\{2,15,23\},\{7,8,14\},\{1,15,20\},\{6,20,23\}$,
$\{7,19,24\},\{3,4,13\},\{6,11,19\},\{22,23,24\},\{3,14,20\},\{8,18,19\}$,
$\{5,17,20\},\{1,18,23\},\{2,13,22\},\{3,8,16\},\{7,9,20\},\{8,17,23\},\{1,8,11\}$,
$\{14,21,23\},\{9,10,19\},\{10,11,23\},\{8,9,21\},\{0,6,17\},\{3,17,24\},\{2,3,6\}$, $\{9,13,23\},\{16,19,20\},\{10,12,21\},\{6,21,22\},\{1,13,17\},\{1,6,9\}$,
$\{4,10,20\},\{0,5,19\},\{13,15,19\},\{12,20,24\},\{2,11,21\},\{0,1,7\},\{1,3,21\}$, $\{0,8,15\},\{6,10,13\},\{4,14,15\},\{0,2,12\},\{3,11,15\},\{15,16,21\},\{4,5,11\}$, $\{0,3,9\},\{2,9,14\},\{17,18,21\},\{1,2,19\},\{18,20,22\},\{9,16,24\},\{7,17,22\}$, $\{4,9,18\},\{7,11,12\},\{2,8,20\},\{6,14,24\},\{5,21,24\},\{11,16,17\},\{4,19,21\}$, $\{2,18,24\},\{11,13,20\},\{5,13,14\},\{9,11,22\},\{0,4,22\},\{0,10,14\}$,
$\{9,15,17\},\{3,19,22\},\{11,14,18\},\{0,16,23\},\{0,20,21\},\{12,19,23\}$, $\{1,5,16\},\{8,13,24\},\{4,6,8\},\{1,10,22\},\{10,15,24\},\{2,5,7\},\{1,12,14\}$, $\{7,13,21\},\{2,4,16\},\{12,13,16\},\{4,12,17\},\{7,15,18\},\{8,12,22\},\{3,5,23\}$, $\{4,7,23\},\{5,15,22\},\{1,4,24\},\{6,12,15\},\{6,7,16\},\{5,6,18\},\{2,10,17\}$, $\{10,16,18\},\{0,13,18\},\{0,11,24\},\{14,16,22\}$
intersection 2 :
$\{1,3,5\},\{5,8,10\}$
$\{6,10,18\},\{13,15,21\},\{7,10,19\},\{0,3,20\},\{6,13,16\},\{3,11,18\}$, $\{9,17,22\},\{2,3,21\},\{10,14,17\},\{16,17,24\},\{2,14,16\},\{0,4,16\}$, $\{16,22,23\},\{1,14,15\},\{9,10,13\},\{15,18,24\},\{0,2,22\},\{5,15,17\}$, $\{3,8,19\},\{5,16,19\},\{8,12,16\},\{0,6,12\},\{13,19,20\},\{4,14,21\},\{6,8,21\}$, $\{5,7,23\},\{7,12,15\},\{4,12,19\},\{0,11,23\},\{1,4,23\},\{6,17,20\},\{6,15,23\}$, $\{0,13,14\},\{11,16,20\},\{0,1,10\},\{1,13,22\},\{8,9,11\},\{18,21,22\},\{4,7,20\}$, $\{5,14,22\},\{3,10,23\},\{3,4,15\},\{9,19,23\},\{10,15,16\},\{3,12,17\}$,
$\{21,23,24\},\{1,8,24\},\{15,19,22\},\{0,7,17\},\{2,8,17\},\{4,10,24\},\{5,12,13\}$, $\{5,20,24\},\{13,17,23\},\{6,11,19\},\{0,9,15\},\{0,19,24\},\{8,15,20\},\{1,9,12\}$, $\{2,5,6\},\{14,18,19\},\{2,10,12\},\{7,9,21\},\{10,20,21\},\{3,9,16\},\{1,16,21\}$,
$\{6,9,24\},\{2,11,15\},\{4,8,13\},\{12,20,22\},\{3,22,24\},\{5,9,18\},\{11,13,24\}$, $\{4,6,22\},\{4,5,11\},\{17,19,21\},\{8,14,23\},\{0,5,21\},\{11,12,21\},\{7,11,14\}$, $\{7,16,18\},\{2,20,23\},\{2,4,9\},\{2,7,24\},\{2,13,18\},\{12,14,24\},\{1,18,20\}$, $\{1,11,17\},\{12,18,23\},\{3,6,14\},\{4,17,18\},\{1,6,7\},\{9,14,20\},\{10,11,22\}$, $\{3,7,13\},\{1,2,19\},\{7,8,22\},\{0,8,18\}$ intersection 1 :
$\{2,3,9\}$
$\{4,9,16\},\{11,15,17\},\{1,2,19\},\{2,6,23\},\{5,9,22\},\{5,6,24\},\{1,3,4\}$, $\{1,13,17\},\{3,12,14\},\{8,13,14\},\{2,14,17\},\{0,5,14\},\{1,5,7\},\{6,16,17\}$, $\{3,10,18\},\{6,11,21\},\{10,14,22\},\{8,18,22\},\{1,6,12\},\{8,16,20\},\{1,9,10\}$, $\{8,15,19\},\{19,22,23\},\{4,8,17\},\{9,12,15\},\{4,14,19\},\{7,15,24\}$, $\{1,14,15\},\{2,5,15\},\{0,2,18\},\{9,11,18\},\{0,23,24\},\{3,16,22\},\{3,15,23\}$, $\{5,8,12\},\{7,9,23\},\{2,20,24\},\{7,10,16\},\{1,8,24\},\{3,11,24\},\{9,19,24\}$, $\{0,7,22\},\{16,21,23\},\{6,15,22\},\{13,16,19\},\{12,18,24\},\{11,12,13\}$, $\{6,9,13\},\{7,13,20\},\{0,9,17\},\{0,12,20\},\{4,13,15\},\{1,11,22\},\{0,4,6\}$, $\{3,5,13\},\{3,17,20\},\{14,16,24\},\{7,17,18\},\{13,18,23\},\{2,12,16\}$, $\{12,21,22\},\{13,21,24\},\{17,19,21\},\{1,16,18\},\{5,11,16\},\{10,15,21\}$, $\{0,3,19\},\{11,19,20\},\{4,5,18\},\{5,20,21\},\{8,9,21\},\{4,12,23\},\{15,18,20\}$, $\{3,6,8\},\{5,10,19\},\{2,10,11\},\{6,7,14\},\{0,8,11\},\{0,15,16\},\{4,20,22\}$, $\{6,18,19\},\{0,10,13\},\{2,4,21\},\{7,12,19\},\{10,12,17\},\{9,14,20\}$,
$\{1,20,23\},\{6,10,20\},\{0,1,21\},\{11,14,23\},\{17,22,24\},\{5,17,23\},\{2,7,8\}$, $\{2,13,22\},\{3,7,21\},\{14,18,21\},\{8,10,23\},\{4,10,24\},\{4,7,11\}$ intersection 0 :
$\{7,10,12\},\{9,20,23\},\{1,8,15\},\{2,5,14\},\{2,4,9\},\{6,12,15\},\{9,15,24\}$, $\{1,16,24\},\{12,13,24\},\{1,9,21\},\{18,20,24\},\{4,8,14\},\{1,2,7\},\{6,7,19\}$, $\{5,8,24\},\{4,15,16\},\{13,16,22\},\{6,14,21\},\{2,13,21\},\{2,10,24\},\{5,9,13\}$,
$\{10,16,21\},\{0,14,20\},\{3,5,17\},\{4,6,20\},\{12,16,19\},\{3,4,13\},\{1,4,19\}$, $\{13,15,17\},\{10,17,18\},\{2,15,23\},\{4,11,24\},\{0,8,12\},\{12,17,20\}$, $\{3,8,21\},\{0,10,15\},\{6,13,18\},\{9,10,11\},\{1,12,23\},\{7,16,18\},\{4,12,21\}$, $\{0,3,18\},\{5,21,23\},\{7,9,14\},\{9,17,22\},\{6,17,24\},\{3,20,22\},\{8,9,19\}$, $\{0,6,9\},\{11,15,20\},\{11,14,16\},\{5,11,12\},\{3,12,14\},\{1,6,10\},\{2,8,18\}$, $\{1,3,11\},\{14,22,23\},\{0,7,23\},\{18,19,23\},\{11,17,23\},\{5,15,19\}$, $\{5,6,22\},\{4,10,23\},\{8,10,22\},\{3,9,16\},\{0,1,22\},\{15,21,22\},\{1,13,20\}$, $\{2,3,6\},\{0,2,11\},\{7,20,21\},\{14,19,24\},\{7,8,17\},\{0,4,17\},\{4,18,22\}$, $\{8,16,20\},\{5,10,20\},\{0,5,16\},\{9,12,18\},\{6,8,11\},\{10,13,14\},\{4,5,7\}$, $\{2,19,20\},\{11,19,22\},\{17,19,21\},\{3,10,19\},\{7,11,13\},\{3,23,24\}$, $\{6,16,23\},\{2,12,22\},\{11,18,21\},\{8,13,23\},\{14,15,18\},\{3,7,15\}$, $\{2,16,17\},\{0,21,24\},\{7,22,24\},\{0,13,19\},\{1,5,18\},\{1,14,17\}$
$\mathbf{B ( b )} \quad I(15,27)=[0,27]$
Consider the following STS(15):
$\{0,1,2\},\{1,12,14\},\{4,7,13\},\{0,3,4\},\{2,3,7\},\{4,9,11\},\{0,5,6\},\{2,4,8\}$, $\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{0,9,10\},\{2,6,14\},\{5,8,12\},\{0,11,12\}$, $\{2,9,12\},\{5,9,13\},\{0,13,14\},\{2,10,13\},\{6,7,12\},\{1,3,5\},\{3,6,9\},\{6,8,13\}$, $\{1,4,6\},\{3,8,11\},\{6,10,11\},\{1,7,9\},\{3,10,14\},\{7,11,14\},\{1,8,10\},\{3,12,13\}$, $\{8,9,14\},\{1,11,13\},\{4,5,14\}$

The $\operatorname{STS}(27)$ given below intersect this system in the indicated number of blocks.
intersection 27:
$\{0,1,2\},\{1,12,14\},\{3,12,13\},\{0,3,4\},\{2,3,7\},\{4,9,11\},\{0,5,6\}$,
$\{1,8,10\},\{8,9,14\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{0,9,10\},\{3,10,14\}$,
$\{5,8,12\},\{1,7,9\},\{4,5,14\},\{5,9,13\},\{0,13,14\},\{2,10,13\},\{6,7,12\}$,
$\{1,3,5\},\{3,6,9\},\{6,8,13\},\{1,4,6\},\{3,8,11\},\{6,10,11\}$
$\{14,16,20\},\{1,13,16\},\{7,18,19\},\{7,14,21\},\{9,16,25\},\{3,18,23\}$,
$\{4,13,18\},\{2,8,19\},\{13,22,26\},\{7,11,15\},\{11,20,23\},\{5,17,24\}$,
$\{7,22,25\},\{11,13,17\},\{1,15,21\},\{3,16,24\},\{11,16,21\},\{14,19,25\}$,
$\{1,17,18\},\{7,20,24\},\{5,16,26\},\{8,16,17\},\{1,11,26\},\{8,22,23\},\{4,7,16\}$,
$\{8,21,26\},\{1,24,25\},\{0,16,18\},\{10,12,21\},\{13,21,24\},\{14,18,26\}$,
$\{7,13,23\},\{10,15,17\},\{11,12,25\},\{7,17,26\},\{10,16,19\},\{9,17,21\}$,
$\{4,21,25\},\{0,17,20\},\{2,16,22\},\{2,14,15\},\{4,8,20\},\{9,19,26\},\{2,4,17\}$,
$\{8,15,24\},\{0,15,25\},\{3,19,21\},\{13,15,19\},\{14,17,23\},\{5,19,22\}$,
$\{5,15,18\},\{9,18,22\},\{3,15,22\},\{2,21,23\},\{6,19,20\},\{6,15,16\}$,
$\{6,23,26\},\{10,22,24\},\{4,19,24\},\{12,17,19\},\{5,20,21\},\{2,12,18\}$,
$\{6,14,24\},\{1,19,23\},\{11,18,24\},\{2,6,25\},\{1,20,22\},\{8,18,25\}$,
$\{4,10,23\},\{12,16,23\},\{11,14,22\},\{3,17,25\},\{6,18,21\},\{6,17,22\}$, $\{0,12,26\},\{4,12,22\},\{4,15,26\},\{0,23,24\},\{9,15,23\},\{2,9,20\}$,
$\{10,25,26\},\{2,24,26\},\{3,20,26\},\{5,23,25\},\{9,12,24\},\{0,11,19\}$,
$\{0,21,22\},\{13,20,25\},\{10,18,20\},\{12,15,20\}$
intersection 26 :
$\{0,1,2\},\{1,12,14\},\{4,7,13\},\{8,9,14\},\{4,5,14\},\{4,9,11\},\{0,5,6\}$, $\{1,7,9\},\{1,8,10\},\{0,7,8\},\{2,5,11\},\{3,10,14\},\{0,9,10\},\{3,12,13\}$, $\{5,8,12\},\{0,11,12\},\{2,9,12\},\{1,11,13\},\{0,13,14\},\{2,10,13\},\{7,11,14\}$, $\{1,3,5\},\{3,6,9\},\{6,8,13\},\{1,4,6\},\{3,8,11\}$
$\{0,17,21\},\{0,22,23\},\{4,8,25\},\{3,4,23\},\{2,15,16\},\{3,15,26\},\{13,18,22\}$, $\{6,11,21\},\{11,16,23\},\{1,18,25\},\{1,16,19\},\{6,7,18\},\{2,6,17\}$, $\{14,15,25\},\{3,7,21\},\{2,21,26\},\{3,16,22\},\{11,20,24\},\{10,23,25\}$, $\{9,19,22\},\{12,24,25\},\{13,16,26\},\{6,14,23\},\{7,15,17\},\{9,16,24\}$, $\{10,21,24\},\{5,13,20\},\{1,17,22\},\{8,16,21\},\{5,15,18\},\{3,18,19\}$, $\{5,9,25\},\{9,18,20\},\{4,16,20\},\{13,23,24\},\{5,19,24\},\{9,13,21\}$,
$\{3,17,20\},\{0,19,20\},\{8,17,19\},\{1,15,24\},\{10,18,26\},\{5,17,23\}$, $\{6,12,19\},\{2,4,22\},\{2,20,23\},\{14,20,21\},\{6,10,20\},\{8,23,26\},\{2,7,24\}$, $\{4,17,24\},\{4,10,19\},\{5,21,22\},\{0,3,24\},\{6,16,25\},\{7,12,16\},\{2,8,18\}$, $\{2,14,19\},\{7,20,25\},\{0,16,18\},\{12,20,22\},\{8,22,24\},\{13,17,25\}$, $\{12,18,23\},\{4,12,26\},\{6,24,26\},\{1,21,23\},\{10,11,15\},\{11,17,18\}$, $\{14,18,24\},\{12,15,21\},\{4,18,21\},\{0,25,26\},\{13,15,19\},\{7,10,22\}$, $\{9,15,23\},\{11,22,25\},\{14,16,17\},\{2,3,25\},\{7,19,23\},\{0,4,15\}$, $\{5,10,16\},\{9,17,26\},\{6,15,22\},\{5,7,26\},\{10,12,17\},\{8,15,20\}$, $\{19,21,25\},\{11,19,26\},\{1,20,26\},\{14,22,26\}$
intersection 25 :
$\{0,1,2\},\{1,12,14\},\{4,7,13\},\{0,3,4\},\{2,3,7\},\{4,9,11\},\{1,8,10\}$, $\{7,11,14\},\{4,10,12\},\{0,7,8\},\{6,10,11\},\{5,7,10\},\{0,9,10\},\{1,7,9\}$, $\{3,10,14\},\{0,11,12\},\{2,9,12\},\{5,9,13\},\{0,13,14\},\{2,10,13\},\{1,11,13\}$, $\{1,3,5\},\{3,8,11\},\{6,8,13\},\{1,4,6\}$
$\{4,15,18\},\{8,20,21\},\{8,14,22\},\{13,17,22\},\{3,22,26\},\{2,6,21\}$, $\{5,19,20\},\{2,4,20\},\{11,23,25\},\{12,13,25\},\{2,17,19\},\{11,18,19\}$, $\{4,8,17\},\{10,15,17\},\{12,17,26\},\{9,19,21\},\{6,7,17\},\{1,21,22\}$, $\{0,15,24\},\{6,15,20\},\{1,17,24\},\{7,16,25\},\{3,12,16\},\{6,12,19\}$,
$\{10,20,23\},\{4,5,24\},\{9,14,18\},\{6,9,23\},\{9,16,17\},\{5,6,22\},\{4,21,25\}$, $\{0,21,23\},\{3,6,25\},\{7,22,23\},\{10,16,26\},\{8,15,25\},\{8,12,18\}$, $\{15,21,26\},\{9,15,22\},\{2,18,23\},\{5,11,21\},\{3,18,21\},\{9,24,25\}$, $\{7,20,26\},\{10,18,22\},\{4,23,26\},\{3,9,20\},\{5,12,15\},\{13,24,26\}$, $\{0,19,22\},\{5,14,25\},\{1,16,18\},\{11,15,16\},\{1,19,26\},\{11,22,24\}$, $\{2,11,26\},\{10,19,25\},\{17,18,25\},\{0,25,26\},\{0,5,17\},\{3,19,24\}$, $\{10,21,24\},\{2,22,25\},\{14,16,23\},\{7,15,19\},\{7,18,24\},\{8,9,26\}$, $\{4,16,22\},\{4,14,19\},\{3,17,23\},\{11,17,20\},\{7,12,21\},\{2,14,15\}$, $\{2,5,16\},\{2,8,24\},\{13,19,23\},\{6,14,26\},\{12,20,22\},\{6,16,24\}$, $\{1,20,25\},\{13,18,20\},\{12,23,24\},\{13,16,21\},\{8,16,19\},\{3,13,15\}$, $\{0,16,20\},\{1,15,23\},\{14,20,24\},\{14,17,21\},\{0,6,18\},\{5,18,26\},\{5,8,23\}$ intersection 24 :
$\{0,1,2\},\{6,10,11\},\{4,7,13\},\{0,3,4\},\{2,3,7\},\{4,9,11\},\{0,5,6\},\{1,4,6\}$, $\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{0,9,10\},\{1,11,13\},\{5,8,12\}$,
$\{0,11,12\},\{2,9,12\},\{5,9,13\},\{3,12,13\},\{2,10,13\},\{6,7,12\},\{3,8,11\}$, $\{7,11,14\},\{3,10,14\}$
$\{8,24,26\},\{9,15,19\},\{3,21,22\},\{10,18,25\},\{12,16,24\},\{2,21,26\}$, $\{6,13,14\},\{0,20,24\},\{1,12,17\},\{4,19,21\},\{7,17,25\},\{4,8,17\},\{6,15,26\}$, $\{7,9,26\},\{1,5,26\},\{6,9,16\},\{3,6,17\},\{2,8,19\},\{1,9,24\},\{1,8,18\}$, $\{4,14,16\},\{7,20,22\},\{4,15,24\},\{1,10,20\},\{4,5,23\},\{12,15,22\}$, $\{5,15,17\},\{9,23,25\},\{11,16,20\},\{1,22,25\},\{4,20,25\},\{14,19,26\}$, $\{0,16,25\},\{6,21,23\},\{11,22,24\},\{3,9,20\},\{2,16,23\},\{14,17,20\}$, $\{5,14,22\},\{1,7,16\},\{18,20,23\},\{11,21,25\},\{7,18,24\},\{1,3,19\}$, $\{12,19,25\},\{7,15,21\},\{3,23,24\},\{2,14,24\},\{11,15,18\},\{2,6,25\}$, $\{11,23,26\},\{2,17,18\},\{6,19,24\},\{9,14,18\},\{12,14,23\},\{7,19,23\}$, $\{1,14,21\},\{5,20,21\},\{0,13,21\},\{12,20,26\},\{0,18,19\},\{1,15,23\}$,
$\{11,17,19\},\{10,21,24\},\{8,13,23\},\{6,8,20\},\{2,4,22\},\{8,10,15\},\{3,5,18\}$, $\{5,16,19\},\{10,16,26\},\{5,24,25\},\{16,17,21\},\{13,19,20\},\{12,18,21\}$, $\{8,9,21\},\{8,14,25\},\{13,15,25\},\{13,17,24\},\{8,16,22\},\{6,18,22\}$, $\{0,22,23\},\{0,17,26\},\{3,25,26\},\{0,14,15\},\{2,15,20\},\{9,17,22\}$, $\{3,15,16\},\{10,17,23\},\{10,19,22\},\{4,18,26\},\{13,22,26\},\{13,16,18\}$ intersection 23 :
$\{1,7,9\},\{1,12,14\},\{4,7,13\},\{0,3,4\},\{1,4,6\},\{4,9,11\},\{0,5,6\}$,
$\{6,10,11\},\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{1,8,10\},\{2,6,14\}$,
$\{6,8,13\},\{1,11,13\},\{2,9,12\},\{5,9,13\},\{7,11,14\},\{2,10,13\},\{6,7,12\}$, $\{1,3,5\},\{4,5,14\}$
$\{5,24,26\},\{13,16,23\},\{7,19,21\},\{10,18,23\},\{5,8,20\},\{0,18,22\}$, $\{7,25,26\},\{1,19,25\},\{12,22,23\},\{11,19,20\},\{4,15,24\},\{5,15,22\}$, $\{3,19,24\},\{0,10,17\},\{11,12,15\},\{2,4,21\},\{6,19,23\},\{0,14,23\}$, $\{1,17,26\},\{2,7,22\},\{8,16,21\},\{12,20,21\},\{2,15,17\},\{0,12,19\}$, $\{8,14,18\},\{21,23,24\},\{2,18,20\},\{9,10,19\},\{8,12,25\},\{3,7,16\}$, $\{7,17,18\},\{6,17,24\},\{14,21,25\},\{8,11,24\},\{1,15,21\},\{5,18,19\}$, $\{2,24,25\},\{5,12,16\},\{10,22,24\},\{14,17,22\},\{6,9,18\},\{13,18,21\}$, $\{10,21,26\},\{12,13,26\},\{2,3,8\},\{14,19,26\},\{0,13,20\},\{4,17,20\}$, $\{11,16,17\},\{10,14,16\},\{13,14,24\},\{0,1,24\},\{16,20,26\},\{3,14,15\}$, $\{3,12,17\},\{4,18,26\},\{3,6,21\},\{0,2,26\},\{8,17,19\},\{3,20,23\},\{3,9,26\}$, $\{3,11,18\},\{1,2,23\},\{5,17,21\},\{6,15,26\},\{0,11,21\},\{10,15,20\}$, $\{13,17,25\},\{9,14,20\},\{12,18,24\},\{9,17,23\},\{15,18,25\},\{8,9,15\}$, $\{1,20,22\},\{6,16,22\},\{11,22,25\},\{13,15,19\},\{4,19,22\},\{3,13,22\}$, $\{4,8,23\},\{5,23,25\},\{6,20,25\},\{9,16,24\},\{4,16,25\},\{0,9,25\},\{8,22,26\}$, $\{9,21,22\},\{2,16,19\},\{3,10,25\},\{7,20,24\},\{1,16,18\},\{7,15,23\}$, $\{11,23,26\},\{0,15,16\}$
intersection 22 :
$\{0,1,2\},\{1,8,10\},\{3,12,13\},\{3,6,9\},\{2,3,7\},\{4,9,11\},\{4,5,14\}$,
$\{7,11,14\},\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{0,9,10\},\{6,8,13\}$,
$\{5,8,12\},\{0,11,12\},\{3,10,14\},\{8,9,14\},\{0,13,14\},\{1,11,13\},\{6,7,12\}$, $\{1,3,5\}$
$\{3,8,18\},\{5,6,25\},\{1,4,18\},\{10,13,24\},\{0,6,15\},\{1,20,21\},\{3,4,19\}$, $\{0,21,24\},\{16,18,25\},\{1,12,22\},\{9,18,26\},\{2,15,25\},\{3,20,24\}$,
$\{2,23,26\},\{2,14,19\},\{5,18,23\},\{7,16,26\},\{1,7,23\},\{5,9,21\},\{10,11,19\}$, $\{15,19,21\},\{7,17,21\},\{4,20,26\},\{2,6,21\},\{10,20,23\},\{3,17,23\}$, $\{0,4,16\},\{19,24,25\},\{2,4,24\},\{11,15,23\},\{8,11,25\},\{2,12,20\}$, $\{10,21,25\},\{5,15,20\},\{12,19,26\},\{1,9,16\},\{6,14,26\},\{8,16,23\}$, $\{14,18,21\},\{7,9,19\},\{8,17,24\},\{9,20,25\},\{2,9,17\},\{13,25,26\}$, $\{14,15,16\},\{16,17,19\},\{6,10,18\},\{14,17,20\},\{2,8,22\},\{8,19,20\}$, $\{12,14,23\},\{8,15,26\},\{2,13,18\},\{1,17,26\},\{0,19,23\},\{14,22,24\}$, $\{5,16,24\},\{4,7,15\},\{10,22,26\},\{12,17,25\},\{13,16,22\},\{0,20,22\}$, $\{6,16,20\},\{9,12,24\},\{4,8,21\},\{11,21,22\},\{11,18,20\},\{12,16,21\}$, $\{0,3,25\},\{1,14,25\},\{7,18,24\},\{12,15,18\},\{1,6,19\},\{4,13,17\}$, $\{10,15,17\},\{3,15,22\},\{9,13,15\},\{7,13,20\},\{5,13,19\},\{2,10,16\}$, $\{4,6,22\},\{4,23,25\},\{3,21,26\},\{5,17,22\},\{13,21,23\},\{0,17,18\}$, $\{6,23,24\},\{1,15,24\},\{7,22,25\},\{0,5,26\},\{3,11,16\},\{18,19,22\}$, $\{11,24,26\},\{9,22,23\},\{6,11,17\}$
intersection 21 :
$\{0,1,2\},\{1,12,14\},\{4,7,13\},\{8,9,14\},\{1,8,10\},\{3,6,9\},\{0,5,6\}$, $\{1,11,13\},\{4,10,12\},\{0,7,8\},\{2,5,11\},\{7,11,14\},\{0,9,10\},\{2,6,14\}$, $\{1,7,9\},\{0,11,12\},\{2,9,12\},\{5,9,13\},\{0,13,14\},\{3,12,13\},\{6,7,12\}$
$\{6,10,24\},\{9,11,20\},\{3,14,25\},\{8,22,23\},\{2,20,24\},\{7,10,19\}$, $\{8,11,16\},\{9,15,21\},\{11,19,24\},\{2,15,23\},\{2,10,25\},\{4,14,22\}$, $\{8,21,25\},\{14,16,24\},\{0,3,20\},\{18,21,24\},\{1,16,26\},\{10,16,17\}$, $\{13,22,26\},\{3,15,24\},\{11,15,22\},\{5,10,14\},\{1,21,23\},\{12,18,22\}$, $\{0,19,21\},\{1,4,15\},\{8,12,19\},\{4,11,25\},\{6,8,26\},\{12,16,21\},\{6,17,18\}$, $\{1,17,24\},\{8,13,24\},\{12,24,25\},\{10,21,26\},\{10,11,18\},\{9,23,25\}$, $\{0,17,26\},\{16,19,25\},\{9,22,24\},\{3,7,17\},\{3,11,26\},\{13,18,19\}$, $\{4,23,26\},\{5,12,26\},\{4,5,18\},\{14,15,18\},\{5,7,21\},\{2,3,16\},\{4,9,17\}$, $\{1,5,25\},\{2,13,21\},\{12,20,23\},\{10,20,22\},\{6,11,21\},\{1,19,20\}$,
$\{0,4,24\},\{7,15,20\},\{14,19,23\},\{13,16,20\},\{3,21,22\},\{6,15,19\}$, $\{4,6,16\},\{17,19,22\},\{2,4,19\},\{10,13,15\},\{13,17,25\},\{1,6,22\}$,
$\{5,17,20\},\{2,7,22\},\{4,20,21\},\{3,10,23\},\{0,18,23\},\{7,24,26\},\{5,23,24\}$,
$\{11,17,23\},\{8,18,20\},\{2,8,17\},\{7,18,25\},\{3,4,8\},\{9,19,26\},\{3,5,19\}$,
$\{0,15,16\},\{6,20,25\},\{7,16,23\},\{6,13,23\},\{9,16,18\},\{12,15,17\}$,
$\{1,3,18\},\{2,18,26\},\{14,17,21\},\{5,16,22\},\{0,22,25\},\{5,8,15\}$,
$\{15,25,26\},\{14,20,26\}$
intersection 20 :
$\{1,7,9\},\{1,12,14\},\{8,9,14\},\{3,12,13\},\{2,3,7\},\{4,9,11\},\{3,10,14\}$,
$\{2,4,8\},\{6,7,12\},\{0,7,8\},\{2,5,11\},\{6,10,11\},\{0,9,10\},\{2,6,14\}$,
$\{5,8,12\},\{0,11,12\},\{1,8,10\},\{3,8,11\},\{1,11,13\},\{1,3,5\}$
$\{8,22,23\},\{4,12,19\},\{10,23,26\},\{5,16,21\},\{7,13,26\},\{12,15,16\}$,
$\{2,19,23\},\{2,13,24\},\{7,14,21\},\{2,9,15\},\{1,23,25\},\{4,10,20\},\{0,15,20\}$,
$\{5,9,23\},\{13,17,21\},\{9,22,26\},\{0,5,17\},\{2,22,25\},\{10,15,17\}$,
$\{7,10,22\},\{4,17,23\},\{6,20,22\},\{0,19,24\},\{13,15,22\},\{7,15,24\}$,
$\{5,18,24\},\{11,22,24\},\{3,17,18\},\{14,20,24\},\{9,18,25\},\{8,19,20\}$,
$\{14,17,22\},\{3,19,22\},\{5,7,20\},\{6,8,25\},\{0,4,13\},\{9,13,19\},\{1,17,19\}$,
$\{12,24,25\},\{7,16,23\},\{0,18,22\},\{12,21,22\},\{4,5,22\},\{10,16,24\}$, $\{7,11,17\},\{4,7,18\},\{2,10,12\},\{2,17,20\},\{1,2,18\},\{11,18,20\},\{8,18,26\}$, $\{0,25,26\},\{8,13,16\},\{1,20,21\},\{6,19,21\},\{4,14,25\},\{0,3,16\}$, $\{11,21,25\},\{0,2,21\},\{1,15,26\},\{2,16,26\},\{16,18,19\},\{9,12,17\}$, $\{3,6,23\},\{1,4,24\},\{16,17,25\},\{11,19,26\},\{11,14,16\},\{13,14,18\}$, $\{3,9,21\},\{6,17,26\},\{6,9,24\},\{5,6,13\},\{14,15,19\},\{21,23,24\},\{7,19,25\}$, $\{10,18,21\},\{9,16,20\},\{3,24,26\},\{13,20,23\},\{6,15,18\},\{8,15,21\}$, $\{0,14,23\},\{3,20,25\},\{12,18,23\},\{8,17,24\},\{1,16,22\},\{11,15,23\}$, $\{3,4,15\},\{12,20,26\},\{5,15,25\},\{10,13,25\},\{5,14,26\},\{5,10,19\}$, $\{4,21,26\},\{4,6,16\},\{0,1,6\}$
intersection 19:
$\{7,11,14\},\{1,12,14\},\{4,7,13\},\{0,3,4\},\{1,3,5\},\{3,8,11\},\{0,5,6\}$, $\{2,4,8\},\{6,10,11\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{1,4,6\},\{3,10,14\}$, $\{5,8,12\},\{1,8,10\},\{2,9,12\},\{2,10,13\},\{0,13,14\}$
$\{5,20,23\},\{7,19,23\},\{5,9,25\},\{3,22,23\},\{15,22,26\},\{11,23,26\}$, $\{0,2,18\},\{8,17,19\},\{3,24,26\},\{19,21,24\},\{2,6,23\},\{2,16,24\},\{4,11,25\}$, $\{2,17,20\},\{4,14,26\},\{1,9,22\},\{7,9,15\},\{8,13,23\},\{10,19,25\},\{2,7,21\}$, $\{13,18,25\},\{6,15,21\},\{14,18,19\},\{10,12,17\},\{8,22,25\},\{10,22,24\}$, $\{3,12,15\},\{6,13,22\},\{1,11,21\},\{7,20,24\},\{10,20,26\},\{0,15,19\}$, $\{5,18,24\},\{23,24,25\},\{9,13,20\},\{17,21,25\},\{0,16,22\},\{16,21,26\}$, $\{9,10,21\},\{4,5,15\},\{11,12,20\},\{5,13,21\},\{2,3,25\},\{5,19,26\},\{1,16,18\}$, $\{4,10,16\},\{10,15,18\},\{6,8,16\},\{0,9,11\},\{7,12,16\},\{8,18,20\},\{3,9,17\}$,
$\{13,15,16\},\{8,9,26\},\{15,20,25\},\{11,16,19\},\{8,15,24\},\{4,21,23\}$, $\{14,16,25\},\{0,12,25\},\{6,12,19\},\{6,17,24\},\{1,7,25\},\{3,16,20\}$, $\{0,17,26\},\{1,15,23\},\{9,16,23\},\{2,14,15\},\{3,6,7\},\{12,18,23\},\{4,17,18\}$, $\{1,2,26\},\{6,25,26\},\{2,19,22\},\{0,1,24\},\{6,14,20\},\{0,20,21\},\{9,14,24\}$, $\{4,12,24\},\{12,13,26\},\{1,19,20\},\{4,20,22\},\{1,13,17\},\{3,18,21\}$, $\{4,9,19\},\{5,14,22\},\{12,21,22\},\{6,9,18\},\{7,17,22\},\{11,18,22\}$, $\{11,13,24\},\{0,10,23\},\{11,15,17\},\{7,18,26\},\{5,16,17\},\{3,13,19\}$, $\{8,14,21\},\{14,17,23\}$
intersection 18:
$\{0,1,2\},\{1,12,14\},\{1,4,6\},\{0,3,4\},\{8,9,14\},\{4,9,11\},\{0,5,6\},\{3,8,11\}$, $\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{1,8,10\},\{2,6,14\},\{7,11,14\}$, $\{6,7,12\},\{2,9,12\},\{0,13,14\}$
$\{2,8,19\},\{1,13,15\},\{3,10,13\},\{0,15,16\},\{5,15,22\},\{3,12,17\},\{4,13,26\}$, $\{10,22,23\},\{7,15,19\},\{5,12,18\},\{20,21,24\},\{1,5,24\},\{4,16,19\}$,
$\{0,22,24\},\{12,21,26\},\{11,15,21\},\{1,11,25\},\{5,17,20\},\{14,19,25\}$, $\{5,14,16\},\{14,15,24\},\{10,11,16\},\{0,20,23\},\{4,14,23\},\{13,17,25\}$, $\{2,20,25\},\{1,23,26\},\{2,3,21\},\{9,22,25\},\{5,19,26\},\{6,9,15\},\{12,16,20\}$, $\{11,23,24\},\{0,17,19\},\{3,5,25\},\{2,7,23\},\{5,8,13\},\{1,3,22\},\{1,9,17\}$, $\{9,16,26\},\{4,15,20\},\{4,8,25\},\{14,18,21\},\{17,18,24\},\{21,23,25\}$, $\{2,16,17\},\{8,17,26\},\{8,18,23\},\{0,9,21\},\{15,17,23\},\{3,7,9\},\{1,18,20\}$, $\{3,15,18\},\{8,21,22\},\{6,11,17\},\{6,13,23\},\{12,13,24\},\{4,17,22\}$,
$\{7,13,18\},\{8,12,15\},\{14,22,26\},\{9,10,18\},\{6,10,21\},\{2,10,24\}$,
$\{16,18,25\},\{11,20,26\},\{6,24,25\},\{12,19,23\},\{1,7,16\},\{6,16,22\}$, $\{4,7,24\},\{8,16,24\},\{0,11,18\},\{5,9,23\},\{3,16,23\},\{1,19,21\},\{10,15,25\}$, $\{4,5,21\},\{3,6,19\},\{11,13,19\},\{10,19,20\},\{2,4,18\},\{2,15,26\},\{7,17,21\}$, $\{6,8,20\},\{18,19,22\},\{7,25,26\},\{3,14,20\},\{11,12,22\},\{0,12,25\}$, $\{9,13,20\},\{10,14,17\},\{6,18,26\},\{0,10,26\},\{7,20,22\},\{2,13,22\}$, $\{13,16,21\},\{3,24,26\},\{9,19,24\}$
intersection 17 :
$\{1,8,10\},\{1,12,14\},\{3,10,14\},\{1,7,9\},\{2,3,7\},\{1,11,13\},\{0,5,6\}$, $\{4,5,14\},\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{1,3,5\},\{2,6,14\}$, $\{3,8,11\},\{8,9,14\},\{2,9,12\}$
$\{5,12,21\},\{10,15,18\},\{2,24,26\},\{8,12,16\},\{6,13,16\},\{6,7,22\}$, $\{9,11,26\},\{12,17,22\},\{2,19,20\},\{15,22,23\},\{2,4,22\},\{0,4,11\}$, $\{0,14,15\},\{14,18,23\},\{16,22,25\},\{4,7,17\},\{5,9,16\},\{14,20,22\}$, $\{17,24,25\},\{10,23,26\},\{3,13,18\},\{0,1,20\},\{6,9,10\},\{10,22,24\}$, $\{7,11,15\},\{12,15,19\},\{7,16,26\},\{4,16,23\},\{6,8,17\},\{4,9,24\},\{1,19,23\}$, $\{10,20,25\},\{5,18,22\},\{6,15,21\},\{8,18,25\},\{11,12,25\},\{0,3,25\}$, $\{9,18,21\},\{2,8,15\},\{8,19,24\},\{1,6,24\},\{10,13,19\},\{3,17,23\}$, $\{11,19,22\},\{4,6,25\},\{12,13,24\},\{2,10,21\},\{11,21,23\},\{0,2,16\}$, $\{1,17,21\},\{8,20,21\},\{4,18,20\},\{5,25,26\},\{5,8,23\},\{0,18,24\},\{9,13,17\}$, $\{5,13,20\},\{3,21,24\},\{1,4,15\},\{15,17,26\},\{1,16,18\},\{11,17,20\}$, $\{5,15,24\},\{1,22,26\},\{16,19,21\},\{13,15,25\},\{2,17,18\},\{5,17,19\}$, $\{9,15,20\},\{16,20,24\},\{10,11,16\},\{6,19,26\},\{7,13,14\},\{7,23,24\}$, $\{14,16,17\},\{14,21,26\},\{9,23,25\},\{0,9,19\},\{14,19,25\},\{3,6,12\}$, $\{4,13,21\},\{0,13,26\},\{8,13,22\},\{4,8,26\},\{0,10,17\},\{3,4,19\},\{1,2,25\}$, $\{0,12,23\},\{3,9,22\},\{2,13,23\},\{3,15,16\},\{7,21,25\},\{6,11,18\},\{3,20,26\}$, $\{11,14,24\},\{0,21,22\},\{12,18,26\},\{6,20,23\},\{7,18,19\},\{7,12,20\}$ intersection 16 :
$\{4,5,14\},\{3,10,14\},\{4,7,13\},\{0,3,4\},\{2,3,7\},\{3,8,11\},\{0,5,6\}$,
$\{7,11,14\},\{6,8,13\},\{1,3,5\},\{2,5,11\},\{3,6,9\},\{0,9,10\},\{0,13,14\}$,
$\{2,10,13\},\{1,11,13\}$
$\{0,2,20\},\{13,17,21\},\{7,9,23\},\{5,8,21\},\{11,20,21\},\{10,18,21\}$, $\{3,12,20\},\{7,8,18\},\{7,20,25\},\{11,18,25\},\{2,12,17\},\{18,23,26\}$, $\{9,19,20\},\{4,9,12\},\{8,16,26\},\{8,24,25\},\{14,22,25\},\{20,23,24\}$, $\{8,9,17\},\{13,18,19\},\{4,20,26\},\{13,16,22\},\{10,15,25\},\{1,6,24\}$, $\{0,17,19\},\{0,15,24\},\{9,13,24\},\{12,13,23\},\{4,18,24\},\{5,13,25\}$, $\{9,11,15\},\{0,21,25\},\{2,8,19\},\{6,19,26\},\{3,18,22\},\{6,20,22\}$, $\{12,14,24\},\{1,4,21\},\{3,16,17\},\{7,10,19\},\{10,16,20\},\{8,10,12\}$, $\{1,14,19\},\{2,16,24\},\{1,10,22\},\{6,7,21\},\{3,13,26\},\{5,15,18\},\{6,16,18\}$, $\{5,7,12\},\{3,15,23\},\{3,19,25\},\{2,4,22\},\{5,10,23\},\{0,7,22\},\{12,25,26\}$, $\{5,9,16\},\{2,9,18\},\{1,7,17\},\{6,11,12\},\{8,14,20\},\{2,21,26\},\{4,11,17\}$, $\{5,19,24\},\{14,17,18\},\{10,17,26\},\{12,15,21\},\{7,15,16\},\{6,15,17\}$,
$\{2,14,15\},\{5,22,26\},\{8,15,22\},\{4,6,10\},\{12,19,22\},\{9,21,22\}$,
$\{17,22,24\},\{1,15,26\},\{7,24,26\},\{0,16,23\},\{4,15,19\},\{10,11,24\}$,
$\{1,9,25\},\{19,21,23\},\{4,16,25\},\{2,6,25\},\{14,16,21\},\{1,2,23\},\{0,12,18\}$,
$\{1,12,16\},\{9,14,26\},\{4,8,23\},\{6,14,23\},\{0,11,26\},\{11,22,23\},\{1,18,20\}$, $\{0,1,8\},\{13,15,20\},\{11,16,19\},\{3,21,24\},\{5,17,20\},\{17,23,25\}$ intersection 15 :
$\{0,1,2\},\{1,12,14\},\{2,10,13\},\{3,6,9\},\{1,11,13\},\{4,9,11\},\{3,10,14\}$, $\{0,11,12\},\{4,10,12\},\{7,11,14\},\{6,7,12\},\{2,9,12\},\{8,9,14\},\{5,9,13\}$, $\{5,8,12\}$
$\{9,17,24\},\{3,16,22\},\{9,18,20\},\{12,21,23\},\{2,6,16\},\{11,19,26\}$, $\{15,17,22\},\{0,3,7\},\{11,18,23\},\{2,14,24\},\{13,17,21\},\{7,10,19\}$, $\{16,19,21\},\{2,3,15\},\{9,21,26\},\{2,11,22\},\{23,24,26\},\{2,18,26\}$,
$\{2,4,19\},\{1,20,22\},\{0,18,21\},\{2,17,25\},\{16,20,23\},\{3,8,13\},\{1,4,23\}$, $\{3,4,20\},\{6,8,22\},\{12,13,25\},\{3,12,19\},\{3,18,25\},\{8,11,21\}$, $\{12,18,22\},\{14,17,18\},\{1,19,24\},\{15,18,19\},\{5,25,26\},\{2,5,23\}$, $\{7,21,25\},\{0,4,17\},\{2,7,8\},\{6,14,19\},\{0,8,26\},\{8,19,25\},\{12,20,24\}$, $\{14,15,26\},\{4,8,18\},\{7,13,24\},\{0,14,20\},\{4,6,26\},\{1,9,25\},\{9,10,22\}$, $\{13,14,23\},\{6,13,18\},\{7,9,16\},\{4,7,22\},\{3,11,24\},\{5,11,15\},\{4,13,15\}$, $\{9,19,23\},\{4,5,16\},\{7,20,26\},\{15,20,25\},\{13,19,20\},\{1,3,26\}$, $\{0,10,25\},\{0,9,15\},\{11,16,17\},\{1,16,18\},\{1,7,17\},\{7,15,23\},\{1,5,10\}$, $\{10,15,21\},\{2,20,21\},\{10,16,26\},\{1,6,21\},\{14,16,25\},\{5,7,18\}$, $\{6,15,24\},\{22,23,25\},\{10,11,20\},\{12,17,26\},\{0,5,24\},\{0,13,16\}$, $\{6,11,25\},\{6,10,17\},\{0,6,23\},\{10,18,24\},\{5,14,22\},\{21,22,24\}$, $\{3,5,21\},\{8,17,20\},\{1,8,15\},\{5,6,20\},\{12,15,16\},\{3,17,23\},\{4,14,21\}$, $\{5,17,19\},\{8,16,24\},\{4,24,25\},\{8,10,23\},\{0,19,22\},\{13,22,26\}$ intersection 14:
$\{1,7,9\},\{1,12,14\},\{4,7,13\},\{1,3,5\},\{5,8,12\},\{7,11,14\},\{6,8,13\}$, $\{2,4,8\},\{6,10,11\},\{0,7,8\},\{0,11,12\},\{2,10,13\},\{3,10,14\},\{2,6,14\}$
$\{15,19,24\},\{0,9,19\},\{5,11,17\},\{1,2,11\},\{6,19,23\},\{1,10,17\}$,
$\{15,22,25\},\{3,23,25\},\{10,15,23\},\{2,3,21\},\{3,6,12\},\{0,10,26\},\{0,1,15\}$, $\{2,12,20\},\{5,7,23\},\{8,11,26\},\{1,13,20\},\{2,9,15\},\{2,7,24\},\{9,10,22\}$,
$\{16,22,24\},\{12,15,17\},\{11,20,23\},\{1,18,26\},\{13,19,22\},\{1,8,22\}$,
$\{12,18,22\},\{7,15,26\},\{5,6,26\},\{17,24,26\},\{8,17,19\},\{13,15,21\}$,
$\{11,21,22\},\{14,19,21\},\{4,16,20\},\{5,9,21\},\{7,17,22\},\{14,16,23\}$,
$\{0,18,21\},\{4,6,22\},\{20,21,24\},\{3,13,24\},\{5,16,19\},\{7,12,21\},\{3,4,26\}$, $\{4,5,15\},\{9,18,25\},\{8,10,25\},\{2,19,26\},\{9,20,26\},\{4,12,23\},\{2,18,23\}$, $\{0,6,20\},\{13,16,17\},\{11,16,18\},\{2,5,22\},\{7,20,25\},\{8,16,21\},\{8,9,23\}$, $\{9,12,24\},\{1,6,21\},\{4,10,21\},\{7,10,16\},\{6,24,25\},\{14,20,22\}$,
$\{0,13,23\},\{8,15,20\},\{17,21,23\},\{3,7,19\},\{1,23,24\},\{0,3,22\},\{0,5,24\}$, $\{11,19,25\},\{5,10,20\},\{3,8,18\},\{22,23,26\},\{5,14,25\},\{0,2,16\}$,
$\{10,12,19\},\{9,11,13\},\{5,13,18\},\{4,9,14\},\{4,11,24\},\{13,14,26\}$,
$\{0,4,25\},\{3,17,20\},\{12,13,25\},\{14,15,18\},\{18,19,20\},\{3,9,16\}$,
$\{8,14,24\},\{4,17,18\},\{10,18,24\},\{1,16,25\},\{21,25,26\},\{3,11,15\}$,
$\{0,14,17\},\{12,16,26\},\{2,17,25\},\{1,4,19\},\{6,9,17\},\{6,7,18\},\{6,15,16\}$ intersection 13 :
$\{0,1,2\},\{3,10,14\},\{1,4,6\},\{5,8,12\},\{2,3,7\},\{1,7,9\},\{0,5,6\},\{2,4,8\}$, $\{4,10,12\},\{0,7,8\},\{2,5,11\},\{5,7,10\},\{5,9,13\}$
$\{9,12,23\},\{15,17,25\},\{3,15,16\},\{11,23,24\},\{1,14,26\},\{0,3,25\}$, $\{4,7,25\},\{3,5,17\},\{7,13,26\},\{6,10,25\},\{4,13,17\},\{1,3,13\},\{7,15,22\}$, $\{8,10,13\},\{9,14,15\},\{3,8,9\},\{0,4,9\},\{2,12,13\},\{16,18,22\},\{13,20,21\}$, $\{9,19,24\},\{21,23,26\},\{6,21,24\},\{14,19,25\},\{7,16,23\},\{2,6,15\}$,
$\{1,17,24\},\{2,9,26\},\{8,15,18\},\{1,16,25\},\{8,11,19\},\{5,16,26\},\{5,14,23\}$, $\{10,19,22\},\{3,11,26\},\{0,12,24\},\{9,10,20\},\{4,11,15\},\{2,21,25\}$, $\{9,16,17\},\{9,22,25\},\{12,15,20\},\{6,13,19\},\{7,11,21\},\{18,19,23\}$, $\{4,20,26\},\{6,7,18\},\{1,15,19\},\{2,10,16\},\{3,6,20\},\{1,8,23\},\{13,14,16\}$, $\{7,12,14\},\{11,13,18\},\{11,16,20\},\{7,17,19\},\{2,17,18\},\{0,16,19\}$, $\{2,14,24\},\{4,21,22\},\{6,9,11\},\{3,19,21\},\{13,23,25\},\{1,10,11\}$, $\{0,14,21\},\{12,17,22\},\{6,8,14\},\{3,4,23\},\{7,20,24\},\{0,20,23\},\{4,16,24\}$, $\{10,24,26\},\{10,15,23\},\{2,19,20\},\{5,18,24\},\{0,11,17\},\{0,15,26\}$, $\{6,17,23\},\{6,22,26\},\{8,17,26\},\{1,5,22\},\{1,18,20\},\{8,16,21\},\{0,10,18\}$, $\{3,22,24\},\{3,12,18\},\{9,18,21\},\{8,24,25\},\{18,25,26\},\{8,20,22\}$,
$\{10,17,21\},\{4,5,19\},\{5,15,21\},\{1,12,21\},\{4,14,18\},\{11,12,25\}$,
$\{12,19,26\},\{6,12,16\},\{0,13,22\},\{13,15,24\},\{2,22,23\},\{11,14,22\}$,
$\{5,20,25\},\{14,17,20\}$
intersection 12 :
$\{0,1,2\},\{3,8,11\},\{3,10,14\},\{0,11,12\},\{2,3,7\},\{1,3,5\},\{1,11,13\}$, $\{2,4,8\},\{4,10,12\},\{3,12,13\},\{1,8,10\},\{2,10,13\}$
$\{4,16,25\},\{13,20,24\},\{1,7,23\},\{13,19,21\},\{2,16,18\},\{0,14,25\}$, $\{9,18,26\},\{4,20,22\},\{4,6,9\},\{2,20,26\},\{4,23,24\},\{1,24,26\},\{7,11,20\}$, $\{15,16,19\},\{14,21,23\},\{2,5,14\},\{2,12,19\},\{3,18,24\},\{6,18,22\}$,
$\{6,10,21\},\{9,23,25\},\{1,18,19\},\{0,21,22\},\{3,19,20\},\{14,15,24\}$, $\{5,6,15\},\{1,15,21\},\{4,11,19\},\{3,17,22\},\{3,15,23\},\{5,16,24\},\{5,19,26\}$, $\{2,21,25\},\{19,22,24\},\{6,7,25\},\{8,23,26\},\{17,21,26\},\{9,11,14\}$, $\{1,16,22\},\{6,14,20\},\{5,7,8\},\{6,11,24\},\{0,3,6\},\{1,6,12\},\{0,9,15\}$,
$\{4,17,18\},\{7,9,19\},\{11,18,25\},\{13,22,23\},\{8,9,21\},\{8,22,25\}$,
$\{7,16,17\},\{11,15,26\},\{0,10,24\},\{5,10,23\},\{9,10,22\},\{4,5,13\},\{1,4,14\}$, $\{3,9,16\},\{4,7,15\},\{8,13,18\},\{0,5,17\},\{12,14,18\},\{3,4,21\},\{10,15,18\}$, $\{2,6,17\},\{5,20,25\},\{9,13,17\},\{5,9,12\},\{6,8,19\},\{6,16,23\},\{18,20,23\}$, $\{13,15,25\},\{12,20,21\},\{8,14,16\},\{8,12,15\},\{14,17,19\},\{0,8,20\}$, $\{1,9,20\},\{7,10,26\},\{10,19,25\},\{0,4,26\},\{5,11,22\},\{1,17,25\},\{0,13,16\}$, $\{12,16,26\},\{10,11,17\},\{7,12,22\},\{0,19,23\},\{12,24,25\},\{10,16,20\}$, $\{15,17,20\},\{2,9,24\},\{12,17,23\},\{7,21,24\},\{11,16,21\},\{6,13,26\}$, $\{0,7,18\},\{5,18,21\},\{3,25,26\},\{8,17,24\},\{2,11,23\},\{7,13,14\},\{2,15,22\}$, $\{14,22,26\}$
intersection 11:
$\{0,1,2\},\{3,6,9\},\{1,11,13\},\{0,3,4\},\{2,3,7\},\{4,9,11\},\{0,5,6\},\{2,4,8\}$, $\{1,8,10\},\{0,11,12\},\{2,5,11\}$
$\{7,15,25\},\{6,10,12\},\{15,19,21\},\{10,16,25\},\{1,14,22\},\{0,14,19\}$,
$\{5,9,16\},\{3,10,26\},\{15,18,24\},\{11,15,26\},\{3,11,19\},\{7,8,12\},\{6,8,17\}$, $\{7,10,14\},\{1,18,25\},\{1,7,21\},\{0,22,24\},\{9,17,23\},\{18,19,22\}$,
$\{7,19,24\},\{16,17,22\},\{11,20,24\},\{2,22,26\},\{5,10,15\},\{9,13,21\}$, $\{4,10,22\},\{14,16,23\},\{1,4,5\},\{8,20,26\},\{14,17,26\},\{7,16,26\}$, $\{8,24,25\},\{3,5,25\},\{5,7,18\},\{21,22,23\},\{16,19,20\},\{6,13,19\}$, $\{3,17,21\},\{9,18,26\},\{1,6,15\},\{5,24,26\},\{11,17,25\},\{4,19,26\}$, $\{4,23,25\},\{5,12,14\},\{1,9,24\},\{1,3,16\},\{11,16,21\},\{7,13,20\},\{2,13,16\}$, $\{0,8,23\},\{2,15,23\},\{9,14,25\},\{4,7,17\},\{2,14,20\},\{6,21,26\},\{6,11,14\}$, $\{5,20,22\},\{12,25,26\},\{1,17,20\},\{4,12,13\},\{3,12,22\},\{0,21,25\}$, $\{13,22,25\},\{2,9,10\},\{10,20,21\},\{3,14,24\},\{2,6,18\},\{13,17,18\}$, $\{5,13,23\},\{0,7,9\},\{1,23,26\},\{0,16,18\},\{8,9,19\},\{8,15,16\},\{4,14,15\}$, $\{12,16,24\},\{3,13,15\},\{7,11,23\},\{2,17,24\},\{3,8,18\},\{4,6,16\},\{6,7,22\}$, $\{5,17,19\},\{3,20,23\},\{0,10,17\},\{6,23,24\},\{1,12,19\},\{10,11,18\}$, $\{8,13,14\},\{4,21,24\},\{2,12,21\},\{10,13,24\},\{5,8,21\},\{14,18,21\}$, $\{9,12,20\},\{10,19,23\},\{0,13,26\},\{8,11,22\},\{0,15,20\},\{6,20,25\}$, $\{2,19,25\},\{4,18,20\},\{9,15,22\},\{12,18,23\},\{12,15,17\}$
intersection 10 :
$\{0,1,2\},\{1,12,14\},\{5,7,10\},\{0,3,4\},\{5,8,12\},\{4,9,11\},\{2,9,12\}$, $\{2,6,14\},\{4,10,12\},\{0,13,14\}$
$\{0,9,25\},\{10,17,20\},\{11,13,18\},\{3,11,17\},\{4,24,26\},\{5,19,24\}$, $\{2,3,22\},\{10,14,24\},\{12,15,18\},\{21,22,24\},\{4,8,14\},\{3,10,13\}$, $\{17,18,19\},\{13,23,26\},\{4,15,23\},\{11,14,21\},\{12,21,25\},\{4,6,18\}$, $\{0,18,22\},\{6,8,22\},\{10,19,21\},\{14,16,18\},\{0,8,17\},\{0,21,26\}$, $\{2,13,19\},\{9,16,23\},\{2,10,25\},\{5,16,21\},\{7,13,16\},\{3,14,23\},\{1,6,7\}$, $\{2,7,15\},\{12,16,24\},\{5,22,23\},\{8,11,15\},\{2,11,24\},\{11,16,26\}$, $\{1,3,26\},\{2,8,20\},\{1,4,22\},\{14,20,22\},\{0,6,24\},\{2,4,16\},\{8,10,23\}$, $\{5,14,25\},\{9,15,26\},\{2,5,26\},\{1,5,17\},\{5,6,15\},\{0,10,16\},\{0,5,11\}$, $\{6,13,17\},\{12,17,23\},\{12,22,26\},\{8,18,24\},\{2,18,23\},\{3,19,20\}$, $\{13,22,25\},\{1,9,20\},\{3,7,18\},\{1,8,19\},\{16,17,22\},\{11,12,19\}$, $\{7,23,25\},\{10,18,26\},\{3,24,25\},\{0,19,23\},\{9,19,22\},\{7,19,26\}$,
$\{11,20,23\},\{2,17,21\},\{12,13,20\},\{9,17,24\},\{3,8,16\},\{15,17,25\}$, $\{7,9,14\},\{16,20,25\},\{3,15,21\},\{1,13,21\},\{3,6,12\},\{0,15,20\}$, $\{14,15,19\},\{3,5,9\},\{1,15,16\},\{7,20,24\},\{9,18,21\},\{4,5,13\},\{1,10,11\}$, $\{1,23,24\},\{7,11,22\},\{4,7,17\},\{6,9,10\},\{8,9,13\},\{14,17,26\},\{5,18,20\}$, $\{10,15,22\},\{0,7,12\},\{4,19,25\},\{6,16,19\},\{6,20,26\},\{6,21,23\}$, $\{13,15,24\},\{7,8,21\},\{4,20,21\},\{6,11,25\},\{8,25,26\},\{1,18,25\}$ intersection 9:
$\{0,13,14\},\{5,8,12\},\{1,11,13\},\{8,9,14\},\{1,3,5\},\{2,6,14\},\{0,5,6\}$, $\{1,4,6\},\{5,7,10\}$
$\{0,11,24\},\{1,9,18\},\{2,4,15\},\{9,21,23\},\{0,1,16\},\{8,18,25\},\{14,17,20\}$, $\{2,5,23\},\{8,20,23\},\{6,18,22\},\{4,9,20\},\{14,18,26\},\{0,4,21\},\{6,9,19\}$, $\{1,21,22\},\{6,13,26\},\{6,8,10\},\{12,22,26\},\{12,14,25\},\{10,16,25\}$,
$\{5,18,21\},\{0,9,26\},\{15,18,19\},\{3,19,25\},\{1,12,15\},\{5,11,17\},\{0,2,7\}$,
$\{3,17,21\},\{10,19,21\},\{4,7,12\},\{4,17,24\},\{2,12,13\},\{0,19,20\}$,
$\{13,18,20\},\{0,10,23\},\{2,8,19\},\{13,23,25\},\{0,15,25\},\{6,11,12\}$,
$\{14,15,21\},\{9,15,22\},\{0,12,18\},\{24,25,26\},\{1,19,24\},\{2,16,24\}$,
$\{4,11,23\},\{3,18,23\},\{11,16,18\},\{4,10,18\},\{1,8,26\},\{2,20,21\}$, $\{3,10,15\},\{8,11,22\},\{5,13,22\},\{3,4,13\},\{11,15,20\},\{10,14,24\}$, $\{0,3,22\},\{13,15,24\},\{15,17,26\},\{4,14,22\},\{4,5,25\},\{16,21,26\}$, $\{22,23,24\},\{9,10,13\},\{2,17,18\},\{10,11,26\},\{7,8,15\},\{10,20,22\}$, $\{13,17,19\},\{6,20,24\},\{2,3,26\},\{3,11,14\},\{1,2,10\},\{5,15,16\},\{3,6,16\}$, $\{11,21,25\},\{7,11,19\},\{1,17,23\},\{3,9,12\},\{7,17,22\},\{3,7,20\}$,
$\{12,21,24\},\{2,9,11\},\{12,16,20\},\{4,19,26\},\{5,20,26\},\{5,14,19\}$, $\{7,9,25\},\{9,16,17\},\{7,23,26\},\{0,8,17\},\{6,17,25\},\{6,15,23\},\{16,19,22\}$, $\{1,20,25\},\{2,22,25\},\{3,8,24\},\{8,13,21\},\{1,7,14\},\{5,9,24\},\{6,7,21\}$, $\{10,12,17\},\{4,8,16\},\{7,13,16\},\{12,19,23\},\{7,18,24\},\{14,16,23\}$ intersection 8:
$\{7,11,14\},\{4,5,14\},\{0,11,12\},\{5,8,12\},\{2,3,7\},\{5,7,10\},\{2,9,12\}$, $\{5,9,13\}$
$\{18,19,23\},\{12,15,25\},\{11,16,25\},\{1,14,26\},\{4,9,19\},\{2,8,22\}$, $\{10,15,24\},\{2,17,19\},\{12,23,24\},\{1,13,16\},\{4,21,22\},\{3,23,25\}$, $\{12,21,26\},\{1,3,4\},\{3,11,17\},\{8,15,21\},\{7,8,26\},\{3,9,26\},\{8,9,16\}$, $\{10,13,20\},\{8,17,18\},\{7,17,24\},\{6,10,19\},\{11,23,26\},\{0,13,15\}$, $\{0,19,21\},\{9,14,24\},\{10,18,26\},\{1,5,11\},\{2,11,21\},\{13,18,21\}$, $\{0,10,17\},\{4,8,24\},\{1,19,22\},\{0,3,20\},\{11,15,20\},\{13,17,26\},\{2,13,23\}$, $\{4,10,25\},\{1,24,25\},\{20,25,26\},\{17,22,23\},\{2,6,25\},\{9,20,21\}$, $\{5,22,26\},\{3,10,16\},\{2,5,16\},\{10,21,23\},\{5,6,17\},\{0,7,22\},\{14,20,22\}$, $\{9,10,22\},\{7,9,15\},\{14,15,17\},\{1,6,9\},\{5,21,24\},\{8,14,23\},\{6,13,14\}$, $\{1,12,18\},\{4,12,17\},\{0,4,26\},\{15,16,18\},\{6,11,22\},\{12,19,20\}$,
$\{3,5,18\},\{1,7,20\},\{0,9,23\},\{5,15,19\},\{10,12,14\},\{4,11,13\},\{16,22,24\}$, $\{2,15,26\},\{9,17,25\},\{1,15,23\},\{6,16,21\},\{1,17,21\},\{12,13,22\},\{0,1,8\}$, $\{2,14,18\},\{6,8,20\},\{4,7,18\},\{7,21,25\},\{0,6,18\},\{0,5,25\},\{3,8,19\}$, $\{7,12,16\},\{3,6,12\},\{2,4,20\},\{4,6,15\},\{9,11,18\},\{8,13,25\},\{18,22,25\}$,
$\{3,15,22\},\{6,7,23\},\{8,10,11\},\{4,16,23\},\{16,17,20\},\{0,14,16\},\{0,2,24\}$, $\{6,24,26\},\{14,19,25\},\{7,13,19\},\{3,13,24\},\{18,20,24\},\{1,2,10\}$, $\{3,14,21\},\{5,20,23\},\{16,19,26\},\{11,19,24\}$
intersection 7 :
$\{6,8,13\},\{1,12,14\},\{2,4,8\},\{1,4,6\},\{2,3,7\},\{0,13,14\},\{8,9,14\}$
$\{13,19,21\},\{3,16,24\},\{6,10,18\},\{0,24,26\},\{9,16,23\},\{10,17,21\}$, $\{1,8,25\},\{5,10,26\},\{2,11,14\},\{5,17,24\},\{2,10,20\},\{4,12,17\},\{0,15,16\}$, $\{9,21,26\},\{10,12,22\},\{6,14,26\},\{7,12,25\},\{6,7,11\},\{7,14,21\}$, $\{7,15,17\},\{12,15,18\},\{2,15,26\},\{7,20,22\},\{12,21,23\},\{5,14,25\}$, $\{17,18,23\},\{4,15,25\},\{7,19,26\},\{3,10,19\},\{5,6,9\},\{9,12,24\},\{1,7,24\}$, $\{1,19,22\},\{1,15,23\},\{0,3,23\},\{0,11,22\},\{1,11,17\},\{8,10,15\}$,
$\{14,18,19\},\{11,18,21\},\{9,17,22\},\{11,23,26\},\{3,5,21\},\{16,19,25\}$,
$\{18,22,26\},\{3,8,26\},\{17,25,26\},\{8,11,24\},\{0,8,12\},\{0,20,25\}$,
$\{9,11,15\},\{14,22,23\},\{3,4,22\},\{1,20,26\},\{4,14,16\},\{7,13,23\}$,
$\{5,12,20\},\{5,16,18\},\{3,11,20\},\{6,15,24\},\{15,21,22\},\{13,15,20\}$,
$\{0,5,7\},\{21,24,25\},\{0,4,21\},\{4,13,26\},\{2,16,17\},\{6,19,23\},\{11,13,16\}$, $\{3,6,12\},\{4,7,9\},\{2,19,24\},\{4,10,23\},\{9,18,20\},\{1,16,21\},\{0,2,18\}$,
$\{3,9,25\},\{14,17,20\},\{6,22,25\},\{2,5,22\},\{8,17,19\},\{0,9,19\},\{0,1,10\}$, $\{3,14,15\},\{11,12,19\},\{13,22,24\},\{3,13,17\},\{5,15,19\},\{4,5,11\}$, $\{10,11,25\},\{1,3,18\},\{4,19,20\},\{6,16,20\},\{7,8,18\},\{5,8,23\},\{12,16,26\}$, $\{4,18,24\},\{1,2,9\},\{20,23,24\},\{8,16,22\},\{2,23,25\},\{2,12,13\},\{7,10,16\}$, $\{1,5,13\},\{10,14,24\},\{8,20,21\},\{2,6,21\},\{0,6,17\},\{13,18,25\},\{9,10,13\}$ intersection 6 :
$\{0,5,6\},\{1,8,10\},\{4,7,13\},\{2,5,11\},\{3,12,13\},\{1,3,5\}$
$\{5,9,15\},\{8,11,21\},\{4,11,20\},\{1,20,25\},\{0,4,9\},\{6,7,11\},\{7,22,24\}$, $\{7,8,23\},\{12,17,22\},\{4,12,26\},\{3,10,20\},\{3,6,18\},\{5,16,23\},\{5,14,24\}$, $\{2,20,23\},\{0,15,22\},\{23,24,26\},\{7,25,26\},\{5,13,25\},\{6,8,15\},\{7,10,18\}$, $\{0,14,23\},\{9,17,26\},\{2,17,18\},\{0,13,21\},\{1,2,12\},\{2,6,24\},\{4,5,22\}$,
$\{3,4,14\},\{5,10,12\},\{5,7,17\},\{3,15,23\},\{11,16,26\},\{11,15,17\},\{2,4,21\}$, $\{13,15,20\},\{10,11,13\},\{13,18,24\},\{8,12,19\},\{8,14,20\},\{9,21,25\}$, $\{8,17,25\},\{2,8,22\},\{9,12,18\},\{0,2,3\},\{3,11,25\},\{1,6,9\},\{12,14,15\}$, $\{2,9,13\},\{18,21,26\},\{9,10,16\},\{6,13,19\},\{6,12,21\},\{5,20,21\}$, $\{3,17,24\},\{4,16,19\},\{11,12,23\},\{15,16,18\},\{15,19,26\},\{11,14,18\}$, $\{17,19,20\},\{1,18,23\},\{13,22,23\},\{9,11,24\},\{6,23,25\},\{10,19,24\}$, $\{0,11,19\},\{4,8,24\},\{16,22,25\},\{9,19,23\},\{3,8,9\},\{2,10,26\},\{19,21,22\}$, $\{1,14,19\},\{6,10,22\},\{7,9,20\},\{15,24,25\},\{3,16,21\},\{2,19,25\},\{4,6,17\}$, $\{1,4,15\},\{1,11,22\},\{10,15,21\},\{5,18,19\},\{0,1,7\},\{4,10,23\},\{10,14,25\}$, $\{0,10,17\},\{6,14,26\},\{4,18,25\},\{0,8,18\},\{7,12,16\},\{2,14,16\},\{0,16,24\}$, $\{1,16,17\},\{2,7,15\},\{0,20,26\},\{8,13,16\},\{3,22,26\},\{12,20,24\}$,
$\{13,14,17\},\{5,8,26\},\{9,14,22\},\{0,12,25\},\{6,16,20\},\{1,13,26\}$, $\{7,14,21\},\{3,7,19\},\{1,21,24\},\{17,21,23\},\{18,20,22\}$
intersection 5 :
$\{0,5,6\},\{4,5,14\},\{8,9,14\},\{2,9,12\},\{6,8,13\}$
$\{7,19,23\},\{11,13,19\},\{1,2,5\},\{13,23,25\},\{2,13,24\},\{5,7,16\},\{1,4,10\}$, $\{12,16,23\},\{1,21,23\},\{3,15,19\},\{1,15,22\},\{4,16,19\},\{6,12,24\}$,
$\{9,11,25\},\{6,14,19\},\{2,3,10\},\{9,19,20\},\{10,15,25\},\{2,7,25\},\{7,9,10\}$, $\{6,9,16\},\{5,22,25\},\{7,13,15\},\{2,14,18\},\{6,17,25\},\{4,11,22\}$,
$\{14,16,17\},\{1,12,18\},\{13,17,20\},\{5,10,18\},\{5,8,23\},\{5,19,21\}$,
$\{6,18,21\},\{10,13,16\},\{7,14,24\},\{2,8,19\},\{4,13,18\},\{1,6,11\},\{8,20,25\}$, $\{15,18,20\},\{1,3,7\},\{0,19,24\},\{2,4,26\},\{5,12,15\},\{3,14,25\},\{1,17,19\}$,
$\{3,5,11\},\{2,21,22\},\{0,3,9\},\{19,25,26\},\{10,17,24\},\{11,17,23\},\{4,6,15\}$, $\{18,19,22\},\{6,7,22\},\{8,11,15\},\{9,18,24\},\{0,12,13\},\{12,14,26\}$, $\{10,11,26\},\{0,7,26\},\{3,4,17\},\{10,14,23\},\{16,18,25\},\{0,10,22\}$, $\{1,9,13\},\{3,13,21\},\{2,6,23\},\{3,18,23\},\{3,16,20\},\{5,9,17\},\{1,24,25\}$, $\{20,22,26\},\{5,13,26\},\{17,21,26\},\{0,2,20\},\{12,21,25\},\{0,15,23\}$, $\{2,15,17\},\{15,16,24\},\{4,20,23\},\{5,20,24\},\{8,18,26\},\{0,1,8\},\{9,22,23\}$, $\{4,7,12\},\{8,16,22\},\{23,24,26\},\{0,16,21\},\{6,10,20\},\{1,14,20\},\{0,4,25\}$, $\{11,12,20\},\{3,8,12\},\{7,20,21\},\{7,11,18\},\{0,11,14\},\{0,17,18\},\{4,8,24\}$, $\{11,21,24\},\{2,11,16\},\{8,10,21\},\{1,16,26\},\{14,15,21\},\{4,9,21\}$, $\{12,17,22\},\{13,14,22\},\{7,8,17\},\{9,15,26\},\{10,12,19\},\{3,6,26\}$, $\{3,22,24\}$
intersection 4:
$\{2,3,7\},\{0,7,8\},\{3,8,11\},\{2,5,11\}$
$\{4,20,24\},\{1,13,16\},\{6,14,17\},\{0,14,19\},\{7,13,21\},\{8,12,21\}$, $\{0,10,13\},\{0,2,26\},\{3,5,17\},\{0,4,6\},\{6,10,12\},\{6,19,20\},\{1,3,21\}$, $\{2,13,20\},\{1,4,14\},\{9,15,26\},\{3,16,23\},\{5,8,20\},\{0,3,20\},\{4,8,18\}$, $\{12,19,26\},\{3,4,25\},\{11,20,21\},\{6,7,18\},\{3,14,26\},\{1,6,22\},\{7,9,10\}$, $\{8,13,22\},\{5,24,25\},\{1,5,18\},\{6,8,23\},\{10,11,14\},\{0,12,18\}$, $\{11,16,26\},\{3,13,24\},\{0,11,17\},\{4,5,10\},\{8,9,17\},\{9,22,25\},\{8,14,16\}$, $\{10,17,21\},\{9,18,21\},\{8,10,26\},\{1,2,10\},\{0,5,9\},\{0,23,25\},\{12,17,25\}$, $\{4,11,15\},\{2,14,23\},\{2,15,21\},\{13,19,25\},\{4,13,26\},\{4,21,22\}$,
$\{11,18,23\},\{15,17,19\},\{2,6,9\},\{2,17,24\},\{9,11,24\},\{19,21,24\}$,
$\{9,13,14\},\{10,20,25\},\{14,18,25\},\{7,12,22\},\{1,8,24\},\{3,10,22\}$,
$\{2,16,25\},\{4,9,16\},\{7,17,26\},\{1,11,12\},\{3,9,12\},\{18,24,26\}$,
$\{13,15,23\},\{7,14,24\},\{7,16,20\},\{5,7,15\},\{15,18,20\},\{14,15,22\}$,
$\{4,17,23\},\{4,7,19\},\{10,19,23\},\{6,11,13\},\{5,12,13\},\{0,1,15\},\{5,16,19\}$,
$\{12,15,16\},\{3,6,15\},\{6,21,25\},\{9,20,23\},\{1,17,20\},\{10,16,18\}$,
$\{5,22,23\},\{3,18,19\},\{13,17,18\},\{1,9,19\},\{7,11,25\},\{5,6,26\},\{0,22,24\}$,
$\{5,14,21\},\{2,18,22\},\{10,15,24\},\{2,4,12\},\{12,14,20\},\{2,8,19\}$,
$\{0,16,21\},\{21,23,26\},\{1,7,23\},\{6,16,24\},\{16,17,22\},\{11,19,22\}$,
$\{20,22,26\},\{12,23,24\},\{1,25,26\},\{8,15,25\}$
intersection 3 :
$\{6,7,12\},\{0,3,4\},\{3,6,9\}$
$\{9,13,20\},\{7,15,23\},\{6,10,25\},\{2,3,23\},\{0,14,18\},\{1,22,23\},\{11,14,21\}$, $\{8,9,16\},\{9,18,25\},\{4,8,24\},\{0,12,20\},\{11,16,24\},\{4,7,25\},\{7,13,18\}$, $\{11,17,23\},\{2,6,24\},\{2,7,26\},\{0,1,15\},\{1,16,20\},\{2,18,21\},\{1,12,17\}$, $\{3,17,21\},\{3,16,19\},\{1,4,9\},\{0,17,26\},\{10,17,24\},\{8,12,25\},\{1,3,24\}$, $\{5,9,24\},\{4,20,23\},\{8,13,26\},\{3,8,14\},\{11,19,25\},\{2,10,20\},\{1,21,26\}$,
$\{2,12,19\},\{4,14,15\},\{10,12,15\},\{8,10,23\},\{1,6,19\},\{17,18,20\}$,
$\{2,4,17\},\{7,9,19\},\{1,7,10\},\{9,12,23\},\{0,5,19\},\{11,20,26\},\{7,16,17\}$, $\{9,10,21\},\{12,13,21\},\{10,19,22\},\{10,13,16\},\{18,19,23\},\{6,11,22\}$, $\{13,14,19\},\{5,16,18\},\{17,22,25\},\{8,19,21\},\{1,8,18\},\{0,10,11\}$, $\{5,7,14\},\{0,21,23\},\{6,15,16\},\{9,11,15\},\{4,5,10\},\{4,6,18\},\{5,20,25\}$, $\{18,22,24\},\{3,7,11\},\{3,10,18\},\{10,14,26\},\{6,20,21\},\{5,8,17\}$,
$\{19,20,24\},\{3,15,20\},\{7,21,22\},\{2,5,22\},\{3,5,12\},\{0,13,25\},\{2,15,25\}$, $\{11,12,18\},\{23,24,26\},\{6,14,23\},\{21,24,25\},\{13,15,24\},\{1,14,25\}$, $\{0,16,22\},\{5,15,21\},\{0,2,9\},\{9,14,17\},\{12,14,24\},\{6,13,17\},\{2,14,16\}$, $\{2,8,11\},\{5,6,26\},\{7,8,20\},\{4,12,22\},\{4,16,21\},\{5,13,23\},\{15,18,26\}$,
$\{3,25,26\},\{15,17,19\},\{16,23,25\},\{0,6,8\},\{0,7,24\},\{4,19,26\},\{9,22,26\}$, $\{12,16,26\},\{14,20,22\},\{4,11,13\},\{1,5,11\},\{8,15,22\},\{3,13,22\},\{1,2,13\}$ intersection 2 :
$\{0,11,12\},\{6,8,13\}$
$\{4,9,15\},\{2,4,6\},\{2,16,21\},\{4,7,8\},\{0,7,15\},\{3,5,7\},\{4,10,26\}$, $\{1,13,15\},\{8,14,18\},\{3,17,22\},\{0,1,21\},\{6,7,10\},\{12,16,17\}$,
$\{11,15,20\},\{19,23,25\},\{1,22,25\},\{10,13,18\},\{8,15,16\},\{0,2,8\}$, $\{3,12,21\},\{9,18,26\},\{7,11,13\},\{14,17,25\},\{13,22,26\},\{9,11,23\}$, $\{1,6,9\},\{4,20,22\},\{10,14,22\},\{9,19,24\},\{1,7,23\},\{9,10,21\},\{10,19,20\}$, $\{5,8,17\},\{14,19,26\},\{8,20,25\},\{8,23,26\},\{8,21,24\},\{5,15,26\},\{5,9,22\}$, $\{8,9,12\},\{20,21,26\},\{2,5,23\},\{3,16,23\},\{3,4,11\},\{4,5,25\},\{5,12,13\}$, $\{0,13,17\},\{12,15,25\},\{6,17,20\},\{7,18,19\},\{7,21,25\},\{10,15,23\}$,
$\{1,5,24\},\{2,7,26\},\{5,11,18\},\{2,3,20\},\{0,3,26\},\{4,13,24\},\{6,16,24\}$, $\{1,2,18\},\{0,16,22\},\{0,9,20\},\{7,20,24\},\{2,10,17\},\{1,16,20\},\{4,16,18\}$, $\{2,15,22\},\{4,12,23\},\{1,17,26\},\{3,6,18\},\{7,14,16\},\{6,14,15\},\{1,3,8\}$, $\{0,23,24\},\{11,17,19\},\{17,18,23\},\{1,10,12\},\{6,12,26\},\{5,6,21\}$, $\{0,6,19\},\{12,14,24\},\{12,18,20\},\{24,25,26\},\{7,12,22\},\{2,12,19\}$, $\{3,15,19\},\{11,21,22\},\{10,16,25\},\{0,5,10\},\{2,11,24\},\{3,9,25\}$, $\{8,19,22\},\{6,22,23\},\{7,9,17\},\{6,11,25\},\{14,21,23\},\{0,18,25\},\{0,4,14\}$, $\{3,10,24\},\{9,13,16\},\{11,16,26\},\{5,16,19\},\{8,10,11\},\{2,9,14\}$, $\{4,17,21\},\{1,4,19\},\{15,18,21\},\{2,13,25\},\{18,22,24\},\{13,20,23\}$, $\{13,19,21\},\{15,17,24\},\{3,13,14\},\{1,11,14\},\{5,14,20\}$
intersection 1 :
$\{7,11,14\}$
$\{19,23,26\},\{8,12,21\},\{1,7,23\},\{2,12,15\},\{0,11,15\},\{2,4,10\}$,
$\{13,16,22\},\{3,17,19\},\{5,15,17\},\{4,5,13\},\{10,14,20\},\{8,10,23\}$,
$\{14,17,21\},\{3,7,25\},\{7,13,26\},\{5,14,23\},\{4,7,17\},\{9,13,21\},\{0,2,23\}$, $\{1,11,26\},\{15,18,26\},\{1,6,12\},\{11,12,24\},\{0,8,13\},\{12,14,26\}$,
$\{0,9,26\},\{3,4,21\},\{2,6,21\},\{2,18,22\},\{3,14,15\},\{14,22,24\},\{17,22,23\}$, $\{3,6,16\},\{12,13,20\},\{5,9,10\},\{5,6,11\},\{11,18,21\},\{1,14,16\}$, $\{11,19,25\},\{10,21,22\},\{4,18,20\},\{16,21,26\},\{16,18,19\},\{10,12,25\}$, $\{0,3,12\},\{5,8,16\},\{17,25,26\},\{6,23,25\},\{1,3,20\},\{20,22,26\},\{0,14,18\}$, $\{0,1,21\},\{0,5,20\},\{2,14,19\},\{6,9,15\},\{5,24,26\},\{8,17,24\},\{2,7,16\}$,
$\{2,13,25\},\{6,8,18\},\{9,12,19\},\{7,15,21\},\{7,8,19\},\{9,11,22\},\{8,14,25\}$, $\{3,13,24\},\{5,19,21\},\{1,2,5\},\{1,10,17\},\{3,18,23\},\{4,11,23\},\{6,13,14\}$, $\{12,16,23\},\{1,18,24\},\{0,10,16\},\{8,15,20\},\{1,4,15\},\{13,15,23\}$,
$\{4,12,22\},\{10,13,18\},\{2,17,20\},\{9,16,17\},\{1,9,25\},\{5,7,12\},\{1,8,22\}$,
$\{6,10,26\},\{15,22,25\},\{3,5,22\},\{0,24,25\},\{20,21,25\},\{19,20,24\}$,
$\{5,18,25\},\{11,16,20\},\{4,6,24\},\{7,10,24\},\{1,13,19\},\{3,10,11\},\{0,6,17\}$, $\{4,9,14\},\{0,7,22\},\{2,8,11\},\{7,9,18\},\{4,8,26\},\{10,15,19\},\{4,16,25\}$,
$\{21,23,24\},\{0,4,19\},\{6,19,22\},\{12,17,18\},\{11,13,17\},\{2,3,26\},\{3,8,9\}$, $\{9,20,23\},\{6,7,20\},\{15,16,24\},\{2,9,24\}$
intersection 0 :
$\{0,2,15\},\{9,11,26\},\{11,15,22\},\{13,23,25\},\{3,10,21\},\{15,16,19\}$, $\{3,5,12\},\{1,15,21\},\{7,21,22\},\{5,15,23\},\{12,17,22\},\{1,2,24\},\{4,7,8\}$, $\{11,17,21\},\{15,18,24\},\{4,5,10\},\{5,7,18\},\{6,8,21\},\{12,14,19\}$, $\{0,20,25\},\{0,4,18\},\{2,10,17\},\{5,24,26\},\{8,18,23\},\{14,18,22\}$, $\{0,10,12\},\{5,8,25\},\{2,4,16\},\{4,14,25\},\{9,17,19\},\{4,20,21\},\{4,11,24\}$, $\{10,18,25\},\{2,22,25\},\{14,16,24\},\{0,13,19\},\{3,24,25\},\{10,19,22\}$,
$\{7,9,25\},\{12,20,23\},\{3,7,26\},\{4,9,15\},\{11,16,18\},\{2,21,23\},\{2,13,18\}$,
$\{5,13,22\},\{20,22,24\},\{6,7,19\},\{3,4,19\},\{0,17,23\},\{4,13,17\},\{0,1,16\}$, $\{6,14,20\},\{3,13,15\},\{12,16,26\},\{10,11,23\},\{0,5,9\},\{8,13,16\}$, $\{3,14,17\},\{0,8,14\},\{18,19,20\},\{1,4,12\},\{8,15,17\},\{4,6,22\},\{12,13,24\}$, $\{6,10,26\},\{0,3,6\},\{5,11,20\},\{0,7,11\},\{9,22,23\},\{2,5,19\},\{2,7,20\}$,
$\{7,17,24\},\{3,8,20\},\{1,3,23\},\{14,15,26\},\{0,21,24\},\{16,21,25\},\{2,8,26\}$, $\{9,13,21\},\{1,7,13\},\{8,10,24\},\{1,9,10\},\{8,11,19\},\{10,15,20\},\{0,22,26\}$, $\{19,23,24\},\{1,8,22\},\{7,10,16\},\{1,11,14\},\{4,23,26\},\{10,13,14\}$,
$\{7,12,15\},\{1,17,20\},\{1,18,26\},\{1,19,25\},\{8,9,12\},\{6,15,25\},\{2,6,12\}$, $\{6,17,18\},\{9,16,20\},\{3,16,22\},\{17,25,26\},\{12,18,21\},\{19,21,26\}$,
$\{2,3,11\},\{6,16,23\},\{3,9,18\},\{5,14,21\},\{6,11,13\},\{7,14,23\},\{2,9,14\}$, $\{13,20,26\},\{6,9,24\},\{1,5,6\},\{5,16,17\},\{11,12,25\}$

## C Cases from Theorem 28:

$$
\begin{gathered}
\mathbf{C}(\mathbf{a}) \quad b-1, b-2, b-3, b-5 \in I(7,15), I(7,19), I(7,21) \\
2,4,5,6 \in I(7,15)
\end{gathered}
$$

$\{2,3,6\}$
$\{0,1,2\},\{0,3,4\},\{0,5,6\},\{1,3,5\},\{1,4,6\},\{2,4,5\}$
$\{2,9,13\},\{0,13,14\},\{5,8,13\},\{1,8,14\},\{4,7,13\},\{6,8,11\},\{5,7,9\}$, $\{7,12,14\},\{2,7,8\},\{0,7,11\},\{3,8,12\},\{0,8,9\},\{1,9,11\},\{6,9,14\}$, $\{5,10,14\},\{0,10,12\},\{1,12,13\},\{1,7,10\},\{3,6,7\},\{5,11,12\},\{3,11,13\}$, $\{4,8,10\},\{2,6,12\},\{3,9,10\},\{6,10,13\},\{2,3,14\},\{2,10,11\},\{4,11,14\}$, $\{4,9,12\}$
$\{0,3,4\},\{0,1,2\}$
$\{2,3,6\},\{2,4,5\},\{0,5,6\},\{1,3,5\},\{1,4,6\}$
$\{5,8,10\},\{0,3,11\},\{1,12,13\},\{0,7,8\},\{4,8,9\},\{1,2,8\},\{7,10,12\}$,
$\{4,11,13\},\{1,7,14\},\{5,13,14\},\{4,10,14\},\{2,10,11\},\{6,10,13\},\{0,9,13\}$, $\{3,9,10\},\{1,9,11\},\{5,7,9\},\{8,11,14\},\{2,7,13\},\{2,9,12\},\{5,11,12\}$,
$\{3,8,13\},\{3,12,14\},\{6,9,14\},\{6,8,12\},\{3,4,7\},\{6,7,11\},\{0,2,14\}$,
$\{0,1,10\},\{0,4,12\}$
$\{1,3,5\},\{0,5,6\},\{0,3,4\}$
$\{0,1,2\},\{1,4,6\},\{2,3,6\},\{2,4,5\}$
$\{1,7,8\},\{6,12,13\},\{5,10,11\},\{0,8,9\},\{0,11,12\},\{5,6,7\},\{9,10,14\}$, $\{2,8,12\},\{4,8,14\},\{3,4,10\},\{0,4,13\},\{2,7,9\},\{0,3,5\},\{3,9,13\}$,
$\{3,8,11\},\{1,3,14\},\{6,8,10\},\{1,10,12\},\{5,8,13\},\{1,5,9\},\{1,11,13\}$, $\{6,9,11\},\{2,11,14\},\{0,7,10\},\{3,7,12\},\{4,9,12\},\{2,10,13\},\{0,6,14\}$, $\{5,12,14\},\{7,13,14\},\{4,7,11\}$
$\{1,3,5\},\{2,4,5\},\{0,5,6\},\{0,1,2\},\{0,3,4\}$
$\{2,3,6\},\{1,4,6\}$
$\{0,1,7\},\{1,2,5\},\{1,9,13\},\{3,4,5\},\{1,8,10\},\{0,3,11\},\{0,5,8\}$, $\{10,11,14\},\{2,8,14\},\{1,11,12\},\{5,11,13\},\{0,4,9\},\{0,6,14\},\{0,10,13\}$, $\{3,8,13\},\{6,12,13\},\{3,10,12\},\{3,7,9\},\{8,9,12\},\{0,2,12\},\{5,9,14\}$, $\{2,7,11\},\{1,3,14\},\{2,9,10\},\{4,8,11\},\{5,7,12\},\{2,4,13\},\{6,9,11\}$, $\{4,12,14\},\{5,6,10\},\{7,13,14\},\{6,7,8\},\{4,7,10\}$

$$
2,4,5,6 \in I(7,19)
$$

$\{0,3,4\}$
$\{0,1,2\},\{2,4,5\},\{0,5,6\},\{1,3,5\},\{1,4,6\},\{2,3,6\}$
$\{5,10,16\},\{5,11,13\},\{0,3,11\},\{9,11,12\},\{5,9,18\},\{0,12,17\},\{1,10,15\}$, $\{0,9,14\},\{8,10,12\},\{5,12,15\},\{2,12,13\},\{9,15,16\},\{4,7,17\},\{6,9,17\}$, $\{3,7,9\},\{13,15,17\},\{3,16,18\},\{2,10,18\},\{14,16,17\},\{7,12,18\}$, $\{7,10,11\},\{0,8,18\},\{2,11,17\},\{8,13,16\},\{11,14,18\},\{6,12,16\},\{2,7,16\}$, $\{3,4,12\},\{3,8,15\},\{3,10,17\},\{5,8,17\},\{1,9,13\},\{0,10,13\},\{6,8,11\}$, $\{4,11,15\},\{6,10,14\},\{5,7,14\},\{1,17,18\},\{1,11,16\},\{4,13,18\},\{2,14,15\}$, $\{4,8,14\},\{6,7,13\},\{1,12,14\},\{3,13,14\},\{4,9,10\},\{0,7,15\},\{0,4,16\}$, $\{6,15,18\},\{1,7,8\},\{2,8,9\}$
$\{0,3,4\},\{2,4,5\}$
$\{0,1,2\},\{2,3,6\},\{0,5,6\},\{1,3,5\},\{1,4,6\}$
$\{0,8,15\},\{1,11,18\},\{10,11,17\},\{5,9,13\},\{5,14,18\},\{15,16,17\},\{4,5,17\}$, $\{6,13,14\},\{0,10,16\},\{1,7,14\},\{7,10,18\},\{3,4,11\},\{0,11,14\},\{4,8,9\}$,
$\{6,16,18\},\{7,8,11\},\{7,13,15\},\{0,3,9\},\{3,7,16\},\{3,10,12\},\{11,12,13\}$,
$\{4,13,16\},\{8,14,17\},\{3,14,15\},\{2,11,16\},\{1,9,17\},\{5,12,16\},\{6,7,17\}$,
$\{9,14,16\},\{2,12,14\},\{0,13,18\},\{3,17,18\},\{4,10,14\},\{2,4,15\},\{1,12,15\}$,
$\{6,10,15\},\{9,15,18\},\{5,8,10\},\{5,11,15\},\{6,8,12\},\{2,13,17\},\{6,9,11\}$,
$\{7,9,12\},\{2,8,18\},\{2,9,10\},\{2,5,7\},\{4,12,18\},\{0,12,17\},\{1,10,13\}$,
$\{0,4,7\},\{1,8,16\},\{3,8,13\}$
$\{0,3,4\},\{0,1,2\},\{2,3,6\}$
$\{1,4,6\},\{2,4,5\},\{0,5,6\},\{1,3,5\}$
$\{3,10,13\},\{0,13,15\},\{8,9,13\},\{12,13,18\},\{4,16,18\},\{3,6,17\},\{0,7,11\}$, $\{4,8,10\},\{1,17,18\},\{4,7,15\},\{6,13,16\},\{0,10,18\},\{6,8,14\},\{9,14,18\}$, $\{7,13,14\},\{5,10,17\},\{0,1,16\},\{6,10,11\},\{1,2,13\},\{1,9,10\},\{5,11,13\}$, $\{5,14,16\},\{5,15,18\},\{6,12,15\},\{9,15,17\},\{3,11,18\},\{7,12,17\},\{0,8,17\}$, $\{5,7,9\},\{2,8,18\},\{1,14,15\},\{4,13,17\},\{2,3,14\},\{1,11,12\},\{4,9,11\}$, $\{8,11,16\},\{11,14,17\},\{2,6,9\},\{0,3,9\},\{3,8,15\},\{9,12,16\},\{2,16,17\}$, $\{3,4,12\},\{0,4,14\},\{3,7,16\},\{1,7,8\},\{6,7,18\},\{10,15,16\},\{0,2,12\}$,
$\{5,8,12\},\{2,11,15\},\{10,12,14\},\{2,7,10\}$
$\{2,3,6\},\{1,3,5\},\{2,4,5\},\{0,3,4\},\{1,4,6\}$
$\{0,1,2\},\{0,5,6\}$
$\{0,4,12\},\{1,6,12\},\{4,6,11\},\{0,11,18\},\{2,4,16\},\{2,8,15\},\{4,14,15\}$, $\{13,15,18\},\{2,7,18\},\{6,7,13\},\{0,7,14\},\{1,8,14\},\{5,8,16\},\{8,12,13\}$, $\{0,3,15\},\{6,8,17\},\{14,17,18\},\{2,9,12\},\{7,9,16\},\{7,10,17\},\{3,4,5\}$, $\{3,8,18\},\{12,16,17\},\{1,15,17\},\{2,6,14\},\{4,7,8\},\{11,14,16\},\{3,6,16\}$, $\{1,16,18\},\{5,12,18\},\{10,12,14\},\{0,13,16\},\{6,9,15\},\{4,13,17\}$, $\{10,15,16\},\{5,11,17\},\{1,7,11\},\{6,10,18\},\{3,10,11\},\{2,5,10\},\{1,4,10\}$, $\{3,7,12\},\{0,9,17\},\{2,11,13\},\{8,9,11\},\{4,9,18\},\{1,5,13\},\{5,7,15\}$, $\{2,3,17\},\{0,8,10\},\{1,3,9\},\{11,12,15\},\{5,9,14\},\{3,13,14\},\{9,10,13\}$

$$
2,4,5,6 \in I(7,21)
$$

$\{0,1,2\}$
$\{2,4,5\},\{0,3,4\},\{0,5,6\},\{1,3,5\},\{1,4,6\},\{2,3,6\}$
$\{5,8,11\},\{6,8,18\},\{0,1,10\},\{0,8,20\},\{8,12,16\},\{7,18,20\},\{6,9,14\}$, $\{6,13,19\},\{1,7,14\},\{12,15,18\},\{0,17,18\},\{4,8,10\},\{3,14,15\},\{9,13,20\}$, $\{3,19,20\},\{2,14,20\},\{6,7,10\},\{11,16,18\},\{6,12,17\},\{3,16,17\}$, $\{9,15,17\},\{4,13,16\},\{3,9,18\},\{5,15,20\},\{8,14,17\},\{1,16,20\},\{2,7,8\}$, $\{10,12,20\},\{4,7,15\},\{0,2,16\},\{0,11,15\},\{5,16,19\},\{1,11,17\},\{7,11,19\}$, $\{1,12,13\},\{7,9,16\},\{1,8,15\},\{4,14,18\},\{2,13,17\},\{4,12,19\},\{5,9,10\}$, $\{6,11,20\},\{6,15,16\},\{10,14,16\},\{0,7,13\},\{4,9,11\},\{5,7,17\},\{3,10,11\}$, $\{0,14,19\},\{10,13,15\},\{3,8,13\},\{3,7,12\},\{2,15,19\},\{5,13,18\}$,
$\{11,13,14\},\{10,17,19\},\{8,9,19\},\{4,17,20\},\{0,9,12\},\{1,18,19\}$,
$\{2,11,12\},\{5,12,14\},\{2,10,18\},\{1,2,9\}$
$\{2,3,6\},\{0,1,2\}$
$\{2,4,5\},\{0,3,4\},\{0,5,6\},\{1,3,5\},\{1,4,6\}$
$\{4,15,16\},\{2,8,14\},\{6,7,14\},\{3,9,15\},\{3,6,12\},\{8,10,18\},\{6,16,20\}$,
$\{7,8,17\},\{1,14,20\},\{9,10,16\},\{0,1,12\},\{0,13,19\},\{0,8,20\},\{1,10,13\}$,
$\{1,9,18\},\{5,15,18\},\{4,8,13\},\{5,9,17\},\{1,15,17\},\{3,8,16\},\{6,9,19\}$,
$\{8,9,12\},\{0,10,11\},\{2,11,15\},\{5,8,11\},\{13,17,20\},\{3,13,18\}$,
$\{12,13,15\},\{4,11,19\},\{2,10,12\},\{13,14,16\},\{0,7,9\},\{6,10,17\}$,
$\{5,14,19\},\{3,11,20\},\{0,14,15\},\{2,9,20\},\{2,18,19\},\{10,15,19\}$,
$\{11,14,17\},\{1,8,19\},\{0,17,18\},\{7,16,18\},\{4,18,20\},\{7,15,20\},\{1,2,7\}$,
$\{9,11,13\},\{2,6,13\},\{4,7,10\},\{5,12,16\},\{2,3,17\},\{3,10,14\},\{4,9,14\}$,
$\{7,11,12\},\{12,19,20\},\{6,8,15\},\{4,12,17\},\{6,11,18\},\{5,10,20\}$,
$\{16,17,19\},\{12,14,18\},\{0,2,16\},\{1,11,16\},\{5,7,13\},\{3,7,19\}$
$\{0,5,6\},\{0,1,2\},\{2,4,5\}$

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$\{2,3,6\},\{0,3,4\},\{1,4,6\},\{1,3,5\}$
$\{5,8,15\},\{1,2,19\},\{1,12,17\},\{0,16,19\},\{6,10,11\},\{7,11,18\},\{8,13,19\}$, $\{2,9,13\},\{2,5,7\},\{4,7,20\},\{0,7,14\},\{6,8,9\},\{10,13,15\},\{0,6,18\}$,
$\{3,15,20\},\{1,7,8\},\{6,14,19\},\{13,18,20\},\{5,12,16\},\{1,16,18\},\{6,7,17\}$,
$\{0,9,17\},\{1,13,14\},\{5,9,11\},\{8,10,12\},\{5,6,20\},\{4,13,17\},\{10,17,20\}$,
$\{6,12,13\},\{1,11,20\},\{2,4,12\},\{14,15,17\},\{8,16,20\},\{11,13,16\}$,
$\{9,10,14\},\{2,10,16\},\{0,2,8\},\{4,14,16\},\{0,5,13\},\{9,19,20\},\{3,11,19\}$,
$\{1,9,15\},\{4,5,10\},\{8,17,18\},\{5,17,19\},\{7,12,15\},\{0,12,20\},\{5,14,18\}$,
$\{3,8,14\},\{4,8,11\},\{4,15,19\},\{4,9,18\},\{12,18,19\},\{0,11,15\},\{3,9,12\}$,
$\{7,10,19\},\{3,16,17\},\{2,15,18\},\{2,14,20\},\{0,1,10\},\{6,15,16\},\{3,10,18\}$, $\{2,11,17\},\{11,12,14\},\{7,9,16\},\{3,7,13\}$
$\{2,3,6\},\{1,4,6\},\{0,1,2\},\{0,5,6\},\{0,3,4\}$
$\{2,4,5\},\{1,3,5\}$
$\{4,11,13\},\{5,11,19\},\{3,8,9\},\{9,10,18\},\{2,8,14\},\{5,6,15\},\{3,7,20\}$, $\{1,10,19\},\{3,10,11\},\{7,12,14\},\{0,6,10\},\{2,12,15\},\{1,6,12\},\{0,1,13\}$, $\{11,12,16\},\{7,8,11\},\{2,10,20\},\{5,7,18\},\{5,16,17\},\{0,12,18\},\{6,11,18\}$, $\{4,6,9\},\{3,13,17\},\{0,8,16\},\{0,3,15\},\{2,3,18\},\{1,11,20\},\{0,14,17\}$, $\{4,8,20\},\{8,12,17\},\{7,9,15\},\{7,16,19\},\{5,8,10\},\{15,19,20\},\{11,14,15\}$, $\{9,12,20\},\{0,4,7\},\{2,6,16\},\{13,14,19\},\{13,16,20\},\{1,4,14\},\{1,2,7\}$, $\{5,9,14\},\{2,11,17\},\{7,10,17\},\{5,12,13\},\{1,8,15\},\{2,9,13\},\{4,18,19\}$, $\{3,12,19\},\{10,14,16\},\{3,4,16\},\{8,13,18\},\{0,2,19\},\{1,17,18\},\{1,9,16\}$, $\{14,18,20\},\{10,13,15\},\{3,6,14\},\{4,15,17\},\{15,16,18\},\{0,9,11\}$, $\{6,8,19\},\{0,5,20\},\{9,17,19\},\{4,10,12\},\{6,7,13\},\{6,17,20\}$
$\mathbf{C}(\mathbf{b}) \quad b-1, b-2, b-3, b-4, b-5, b-7 \in I(9,19), I(9,21), I(9,25), I(9,27)$

$$
5,7,8,9,10,11 \in I(9,19)
$$

## $\{2,5,8\}$

$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,4,6\},\{0,4,8\},\{1,5,6\}$, $\{2,3,7\},\{0,5,7\},\{1,3,8\}$
$\{4,10,17\},\{10,15,18\},\{8,9,16\},\{3,11,18\},\{1,12,15\},\{3,9,17\},\{8,10,11\}$, $\{3,13,16\},\{0,9,13\},\{5,8,18\},\{2,13,14\},\{7,11,14\},\{5,13,15\},\{1,14,16\}$, $\{2,9,18\},\{2,5,10\},\{2,8,15\},\{3,14,15\},\{0,10,14\},\{4,15,16\},\{2,12,16\}$, $\{8,13,17\},\{1,9,10\},\{7,13,18\},\{4,12,13\},\{4,9,11\},\{4,14,18\},\{0,16,18\}$, $\{3,10,12\},\{7,10,16\},\{7,9,15\},\{5,14,17\},\{2,11,17\},\{6,11,15\},\{5,9,12\}$, $\{6,10,13\},\{7,12,17\},\{6,16,17\},\{0,15,17\},\{6,9,14\},\{5,11,16\},\{1,17,18\}$, $\{0,11,12\},\{1,11,13\},\{6,12,18\},\{8,12,14\}$
$\{0,5,7\},\{1,5,6\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{0,4,8\},\{1,3,8\}$,
$\{2,3,7\},\{2,4,6\}$
$\{4,13,15\},\{4,9,16\},\{3,16,17\},\{2,11,13\},\{7,16,18\},\{8,9,11\},\{2,10,12\}$, $\{8,13,17\},\{1,12,17\},\{6,17,18\},\{6,9,13\},\{4,10,17\},\{4,11,14\},\{1,5,9\}$, $\{0,5,17\},\{5,6,12\},\{7,10,15\},\{3,10,13\},\{2,9,18\},\{1,11,16\},\{8,10,18\}$, $\{0,11,12\},\{0,15,18\},\{0,13,16\},\{8,14,15\},\{7,9,12\},\{2,14,17\},\{3,12,15\}$, $\{7,11,17\},\{3,11,18\},\{1,13,18\},\{3,9,14\},\{8,12,16\},\{9,15,17\},\{4,12,18\}$, $\{6,10,11\},\{2,15,16\},\{0,9,10\},\{1,6,15\},\{5,7,13\},\{5,11,15\},\{5,10,16\}$, $\{1,10,14\},\{0,7,14\},\{6,14,16\},\{5,14,18\},\{12,13,14\}$
$\{2,5,8\},\{2,3,7\},\{2,4,6\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{0,5,7\},\{0,4,8\},\{1,5,6\}$, $\{1,3,8\}$
$\{2,5,11\},\{1,10,15\},\{6,11,13\},\{5,9,13\},\{2,13,17\},\{2,4,14\},\{6,12,15\}$, $\{8,9,17\},\{0,14,16\},\{1,9,11\},\{3,15,17\},\{7,11,18\},\{7,12,13\},\{2,8,12\}$, $\{4,11,16\},\{1,12,17\},\{4,6,17\},\{5,8,14\},\{4,9,18\},\{9,12,16\},\{3,10,11\}$, $\{8,10,16\},\{2,6,16\},\{0,10,13\},\{7,10,14\},\{8,13,18\},\{4,10,12\},\{3,12,14\}$, $\{3,7,9\},\{2,7,15\},\{3,13,16\},\{1,13,14\},\{6,10,18\},\{14,15,18\},\{6,9,14\}$, $\{1,16,18\},\{0,9,15\},\{8,11,15\},\{0,17,18\},\{7,16,17\},\{2,3,18\},\{4,13,15\}$, $\{5,12,18\},\{5,15,16\},\{0,11,12\},\{5,10,17\},\{11,14,17\},\{2,9,10\}$ $\{2,3,7\},\{0,3,6\},\{3,4,5\},\{2,5,8\}$
$\{0,1,2\},\{0,5,7\},\{6,7,8\},\{1,3,8\},\{1,4,7\},\{2,4,6\},\{0,4,8\},\{1,5,6\}$
$\{7,9,15\},\{7,12,17\},\{13,14,18\},\{0,10,12\},\{2,13,15\},\{1,12,14\}$, $\{11,13,17\},\{3,12,18\},\{0,11,16\},\{3,7,13\},\{3,4,15\},\{1,9,13\},\{2,3,10\}$, $\{0,3,9\},\{6,14,16\},\{9,10,17\},\{0,6,13\},\{5,9,14\},\{1,15,18\},\{6,9,18\}$, $\{8,12,13\},\{3,5,16\},\{0,14,15\},\{0,17,18\},\{7,10,16\},\{3,11,14\},\{6,10,15\}$, $\{6,11,12\},\{5,15,17\},\{5,8,18\},\{7,11,18\},\{3,6,17\},\{2,7,14\},\{1,16,17\}$, $\{2,16,18\},\{8,10,14\},\{12,15,16\},\{2,8,17\},\{8,9,16\},\{2,9,11\},\{4,9,12\}$, $\{4,10,18\},\{5,10,13\},\{4,13,16\},\{8,11,15\},\{1,10,11\},\{2,5,12\},\{4,5,11\}$, $\{4,14,17\}$
$\{0,3,6\},\{3,4,5\},\{2,3,7\},\{0,4,8\},\{2,4,6\}$
$\{0,1,2\},\{1,3,8\},\{6,7,8\},\{1,5,6\},\{1,4,7\},\{2,5,8\},\{0,5,7\}$
$\{0,8,15\},\{3,11,17\},\{4,5,11\},\{8,13,17\},\{13,15,16\},\{6,9,13\},\{7,10,17\}$, $\{6,12,17\},\{2,12,15\},\{4,8,9\},\{5,9,18\},\{0,4,10\},\{2,17,18\},\{5,13,14\}$, $\{0,3,13\},\{6,11,14\},\{8,11,16\},\{7,12,13\},\{7,14,15\},\{0,14,16\},\{1,11,13\}$, $\{9,11,15\},\{1,12,14\},\{9,12,16\},\{7,11,18\},\{8,10,12\},\{4,14,17\},\{4,6,16\}$, $\{3,4,15\},\{5,15,17\},\{2,10,11\},\{2,7,16\},\{6,10,15\},\{3,10,14\},\{8,14,18\}$,
$\{2,9,14\},\{1,16,17\},\{1,9,10\},\{0,11,12\},\{0,9,17\},\{5,10,16\},\{2,3,6\}$,
$\{2,4,13\},\{3,7,9\},\{0,6,18\},\{3,5,12\},\{10,13,18\},\{1,15,18\},\{4,12,18\}$, $\{3,16,18\}$
$\{0,5,7\},\{0,1,2\},\{6,7,8\},\{1,3,8\},\{3,4,5\},\{2,3,7\},\{1,5,6\}$

$$
\{0,4,8\},\{2,5,8\},\{2,4,6\},\{0,3,6\},\{1,4,7\}
$$

$\{2,12,16\},\{9,11,17\},\{1,5,17\},\{10,13,15\},\{0,14,17\},\{8,14,16\},\{5,9,12\}$, $\{1,9,14\},\{1,11,15\},\{3,13,14\},\{3,10,12\},\{4,10,18\},\{1,8,13\},\{6,13,17\}$, $\{6,14,15\},\{6,9,10\},\{3,5,7\},\{5,6,11\},\{1,2,10\},\{7,10,17\},\{4,16,17\}$, $\{4,11,14\},\{3,4,9\},\{2,3,15\},\{6,7,18\},\{2,7,14\},\{0,1,12\},\{4,12,13\}$, $\{12,14,18\},\{7,13,16\},\{2,17,18\},\{9,13,18\},\{4,5,15\},\{6,8,12\},\{3,11,16\}$, $\{12,15,17\},\{5,10,14\},\{0,11,18\},\{7,11,12\},\{1,3,18\},\{2,11,13\}$, $\{5,16,18\},\{3,8,17\},\{7,8,9\},\{0,7,15\},\{0,5,13\},\{8,10,11\},\{0,2,9\}$, $\{8,15,18\},\{0,10,16\},\{9,15,16\},\{1,6,16\}$

$$
5,7,8,9,10,11 \in I(9,21)
$$

$\{2,5,8\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,4,6\},\{0,4,8\},\{1,5,6\}$, $\{2,3,7\},\{0,5,7\},\{1,3,8\}$
$\{7,13,14\},\{9,12,20\},\{5,10,20\},\{14,17,18\},\{5,12,17\},\{7,12,19\}$, $\{8,9,10\},\{5,8,13\},\{0,18,19\},\{7,9,11\},\{4,10,15\},\{8,17,20\},\{3,14,20\}$, $\{1,12,18\},\{1,11,17\},\{3,16,17\},\{1,9,16\},\{0,10,11\},\{3,13,19\},\{2,5,9\}$, $\{0,15,17\},\{6,16,20\},\{3,9,18\},\{2,15,20\},\{0,12,16\},\{3,11,15\},\{2,10,16\}$, $\{2,17,19\},\{6,10,18\},\{2,11,18\},\{7,15,16\},\{11,13,16\},\{4,9,17\},\{0,9,14\}$, $\{10,14,19\},\{1,14,15\},\{5,15,18\},\{1,19,20\},\{1,10,13\},\{2,12,13\}$, $\{6,9,19\},\{5,14,16\},\{6,12,15\},\{6,13,17\},\{3,10,12\},\{4,16,19\},\{7,18,20\}$, $\{2,8,14\},\{8,16,18\},\{5,11,19\},\{4,12,14\},\{6,11,14\},\{9,13,15\},\{8,11,12\}$, $\{0,13,20\},\{8,15,19\},\{7,10,17\},\{4,11,20\},\{4,13,18\}$ $\{0,5,7\},\{3,4,5\}$
$\{0,1,2\},\{1,3,8\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{0,4,8\},\{1,5,6\}$, $\{2,3,7\},\{2,4,6\}$
$\{13,15,16\},\{3,9,20\},\{1,10,13\},\{8,13,17\},\{7,11,14\},\{6,14,17\}$,
$\{5,10,12\},\{8,12,20\},\{0,16,17\},\{8,10,14\},\{14,15,20\},\{2,17,20\}$,
$\{1,18,19\},\{2,12,16\},\{4,5,15\},\{1,9,15\},\{2,13,19\},\{0,10,20\},\{7,10,19\}$, $\{1,11,17\},\{9,14,16\},\{10,17,18\},\{0,11,12\},\{4,19,20\},\{6,12,13\}$,
$\{5,18,20\},\{6,11,20\},\{0,7,9\},\{4,16,18\},\{9,11,13\},\{0,5,14\},\{5,9,17\}$,
$\{2,14,18\},\{7,15,17\},\{8,9,19\},\{3,10,16\},\{1,12,14\},\{8,15,18\},\{3,12,15\}$, $\{3,4,17\},\{7,12,18\},\{6,10,15\},\{4,9,12\},\{3,5,13\},\{6,16,19\},\{4,13,14\}$,
$\{0,13,18\},\{0,15,19\},\{5,7,16\},\{2,11,15\},\{3,11,18\},\{12,17,19\},\{2,9,10\}$, $\{3,14,19\},\{6,9,18\},\{8,11,16\},\{4,10,11\},\{5,11,19\},\{1,16,20\},\{7,13,20\}$ $\{0,4,8\},\{6,7,8\},\{1,5,6\}$
$\{0,1,2\},\{3,4,5\},\{1,3,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{2,4,6\},\{0,5,7\}$, $\{2,3,7\}$
$\{4,9,20\},\{5,18,19\},\{2,11,14\},\{8,14,17\},\{5,16,17\},\{5,12,15\},\{8,13,18\}$, $\{3,12,13\},\{1,14,19\},\{3,19,20\},\{6,7,9\},\{3,11,16\},\{1,10,15\},\{1,6,20\}$, $\{7,12,17\},\{10,18,20\},\{0,8,9\},\{9,11,15\},\{2,12,19\},\{9,17,19\},\{0,13,14\}$, $\{3,14,15\},\{7,10,19\},\{13,15,20\},\{1,16,18\},\{1,9,12\},\{8,10,12\},\{6,8,19\}$, $\{2,10,16\},\{4,15,19\},\{0,4,18\},\{2,17,20\},\{5,9,10\},\{6,17,18\},\{9,14,16\}$, $\{5,11,20\},\{2,15,18\},\{4,13,17\},\{11,13,19\},\{4,12,16\},\{7,14,20\}$, $\{0,12,20\},\{7,11,18\},\{1,5,13\},\{0,15,17\},\{7,13,16\},\{3,10,17\},\{8,16,20\}$, $\{5,6,14\},\{3,9,18\},\{0,16,19\},\{1,11,17\},\{4,8,11\},\{6,10,13\},\{4,10,14\}$, $\{7,8,15\},\{2,9,13\},\{6,15,16\},\{0,10,11\},\{12,14,18\},\{6,11,12\}$ $\{2,3,7\},\{0,1,2\},\{1,3,8\},\{3,4,5\}$
$\{0,5,7\},\{2,4,6\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{0,4,8\},\{1,5,6\}$
$\{3,11,19\},\{2,11,18\},\{8,10,17\},\{2,3,10\},\{1,11,14\},\{4,5,9\},\{5,14,20\}$, $\{4,12,15\},\{7,10,13\},\{0,10,18\},\{4,10,14\},\{1,9,19\},\{5,10,19\},\{1,10,15\}$, $\{1,3,18\},\{13,17,20\},\{1,2,16\},\{0,14,17\},\{10,12,16\},\{4,18,20\}$, $\{6,18,19\},\{15,16,20\},\{7,15,19\},\{2,9,20\},\{5,12,13\},\{9,16,17\}$, $\{5,16,18\},\{3,8,20\},\{7,14,16\},\{3,5,17\},\{0,11,12\},\{3,4,16\},\{7,9,18\}$, $\{3,13,15\},\{2,12,14\},\{0,2,15\},\{4,11,13\},\{1,12,17\},\{15,17,18\}$, $\{9,10,11\},\{2,13,19\},\{13,14,18\},\{8,11,16\},\{5,11,15\},\{6,10,20\}$, $\{6,11,17\},\{6,13,16\},\{3,7,12\},\{7,11,20\},\{6,14,15\},\{6,9,12\},\{8,12,18\}$, $\{3,9,14\},\{1,8,13\},\{12,19,20\},\{8,9,15\},\{4,17,19\},\{0,1,20\},\{8,14,19\}$, $\{2,7,17\},\{0,16,19\},\{0,9,13\}$ $\{1,5,6\},\{6,7,8\},\{0,3,6\},\{2,4,6\},\{0,4,8\}$
$\{0,1,2\},\{3,4,5\},\{1,3,8\},\{0,5,7\},\{1,4,7\},\{2,5,8\},\{2,3,7\}$
$\{6,9,12\},\{7,14,20\},\{0,10,15\},\{8,9,15\},\{8,12,20\},\{7,12,13\},\{9,11,18\}$, $\{15,17,18\},\{10,19,20\},\{1,11,17\},\{3,10,17\},\{2,11,12\},\{4,9,16\}$, $\{0,4,13\},\{8,13,19\},\{5,17,20\},\{11,13,20\},\{3,14,15\},\{6,8,11\},\{3,12,18\}$, $\{0,16,19\},\{4,10,12\},\{2,14,17\},\{8,10,16\},\{2,9,20\},\{7,15,16\},\{2,6,15\}$, $\{1,15,20\},\{0,3,11\},\{0,12,17\},\{2,10,13\},\{5,13,15\},\{10,14,18\},\{2,4,19\}$, $\{5,6,16\},\{5,12,14\},\{3,16,20\},\{5,10,11\},\{3,9,13\},\{1,5,9\},\{5,18,19\}$, $\{0,8,18\},\{7,9,10\},\{0,6,20\},\{13,16,17\},\{4,11,15\},\{7,8,17\},\{4,8,14\}$, $\{0,9,14\},\{1,13,18\},\{1,14,19\},\{7,11,19\},\{1,6,10\},\{9,17,19\},\{6,7,18\}$, $\{2,16,18\},\{1,12,16\},\{4,6,17\},\{4,18,20\},\{12,15,19\},\{3,6,19\}$, $\{11,14,16\},\{6,13,14\}$
$\{1,3,8\},\{1,4,7\},\{0,3,6\},\{0,4,8\},\{3,4,5\},\{0,5,7\},\{2,3,7\}$
$\{0,1,2\},\{1,5,6\},\{6,7,8\},\{2,5,8\},\{2,4,6\}$
$\{8,11,18\},\{4,9,13\},\{2,10,11\},\{0,16,20\},\{3,17,18\},\{3,11,15\},\{2,7,14\}$, $\{9,10,18\},\{5,15,19\},\{6,13,14\},\{14,15,17\},\{4,10,17\},\{5,18,20\}$,
$\{4,7,11\},\{4,5,16\},\{5,12,13\},\{3,6,9\},\{8,9,15\},\{5,7,9\},\{3,7,16\}$,
$\{2,13,16\},\{12,17,19\},\{6,10,15\},\{6,19,20\},\{0,3,4\},\{1,4,18\},\{8,10,16\}$,
$\{1,10,20\},\{4,8,20\},\{6,12,18\},\{2,15,18\},\{13,15,20\},\{9,16,19\}$, $\{14,16,18\},\{0,10,13\},\{8,13,17\},\{3,5,10\},\{7,13,18\},\{0,6,11\}$, $\{11,12,16\},\{3,8,12\},\{1,8,19\},\{1,9,14\},\{7,17,20\},\{5,11,14\},\{9,11,20\}$, $\{1,15,16\},\{4,12,15\},\{6,16,17\},\{0,8,14\},\{4,14,19\},\{10,12,14\},\{0,7,15\}$, $\{1,11,17\},\{0,5,17\},\{2,9,17\},\{1,3,13\},\{3,14,20\},\{1,7,12\},\{2,3,19\}$, $\{0,9,12\},\{2,12,20\},\{0,18,19\},\{7,10,19\},\{11,13,19\}$

$$
5,7,8,9,10,11 \in I(9,25)
$$

$\{0,4,8\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{2,4,6\},\{1,5,6\}$, $\{2,3,7\},\{0,5,7\},\{1,3,8\}$
$\{0,19,23\},\{5,16,20\},\{4,11,21\},\{7,10,21\},\{3,21,22\},\{6,20,22\},\{8,15,16\}$, $\{1,10,15\},\{4,8,24\},\{4,9,13\},\{3,14,20\},\{7,9,12\},\{4,12,16\},\{17,18,24\}$, $\{13,15,21\},\{8,14,18\},\{1,16,19\},\{8,12,19\},\{4,15,20\},\{3,9,11\}$, $\{7,13,17\},\{11,22,23\},\{0,9,16\},\{20,21,24\},\{0,10,24\},\{8,10,20\}$, $\{4,10,19\},\{0,4,14\},\{9,15,17\},\{3,10,16\},\{2,10,22\},\{0,13,20\},\{8,11,17\}$, $\{2,9,23\},\{8,9,22\},\{4,18,22\},\{18,19,21\},\{11,16,24\},\{1,23,24\}$, $\{7,16,22\},\{6,9,24\},\{1,9,14\},\{0,11,12\},\{3,13,18\},\{3,15,23\},\{0,17,22\}$, $\{14,22,24\},\{1,17,21\},\{7,15,24\},\{1,11,20\},\{2,12,24\},\{1,13,22\}$, $\{0,8,21\},\{7,14,23\},\{9,10,18\},\{2,11,13\},\{2,15,19\},\{5,18,23\},\{6,10,23\}$, $\{5,12,14\},\{3,12,17\},\{7,18,20\},\{6,14,15\},\{7,11,19\},\{1,12,18\}$,
$\{10,11,14\},\{9,19,20\},\{12,20,23\},\{13,14,19\},\{3,19,24\},\{6,12,21\}$, $\{5,19,22\},\{6,13,16\},\{10,12,13\},\{4,17,23\},\{2,17,20\},\{6,11,18\}$, $\{2,14,21\},\{8,13,23\},\{0,15,18\},\{5,9,21\},\{2,16,18\},\{12,15,22\}$, $\{16,21,23\},\{6,17,19\},\{5,11,15\},\{14,16,17\},\{5,13,24\},\{5,10,17\}$ $\{2,3,7\},\{0,4,8\}$
$\{0,1,2\},\{3,4,5\},\{6,7,8\},\{0,3,6\},\{1,4,7\},\{2,5,8\},\{1,3,8\},\{1,5,6\}$,
$\{2,4,6\},\{0,5,7\}$
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$\{15,19,22\},\{6,13,23\},\{8,17,18\},\{4,16,18\},\{5,20,23\},\{6,11,16\}$, $\{2,14,22\},\{1,8,15\},\{5,12,24\},\{17,23,24\}$

$$
5,7,8,9,10,11 \in I(9,25)
$$

$\{0,3,6\}$
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$\{11,13,21\},\{7,16,20\},\{14,23,25\},\{6,8,11\},\{12,13,16\},\{2,11,12\}$
$\{0,1,2\},\{0,5,7\},\{0,3,6\},\{6,7,8\},\{0,4,8\}$
$\{2,4,6\},\{3,4,5\},\{2,3,7\},\{1,3,8\},\{1,4,7\},\{2,5,8\},\{1,5,6\}$
$\{3,12,26\},\{0,4,12\},\{7,20,22\},\{3,9,23\},\{5,14,26\},\{13,19,24\},\{7,12,19\}$, $\{1,22,25\},\{1,11,19\},\{6,10,13\},\{16,17,25\},\{6,12,15\},\{5,17,23\},\{6,7,11\}$, $\{0,9,11\},\{3,13,15\},\{4,8,18\},\{14,21,24\},\{4,10,14\},\{5,7,15\},\{12,23,24\}$, $\{0,13,14\},\{8,12,13\},\{15,22,23\},\{4,15,26\},\{2,14,23\},\{16,20,26\}$, $\{9,12,20\},\{15,18,20\},\{19,25,26\},\{1,12,16\},\{3,6,24\},\{2,16,24\}$,
$\{0,3,17\},\{13,18,21\},\{8,11,14\},\{4,11,22\},\{10,12,21\},\{10,15,19\}$,
$\{0,1,15\},\{0,18,22\},\{11,20,24\},\{7,9,21\},\{6,18,19\},\{6,16,23\},\{2,13,20\}$,
$\{1,18,23\},\{7,14,16\},\{11,15,16\},\{9,10,16\},\{17,19,22\},\{6,8,17\}$,
$\{8,23,26\},\{5,16,19\},\{11,12,17\},\{6,9,26\},\{5,10,22\},\{14,15,17\}$,
$\{5,11,13\},\{2,12,22\},\{4,17,20\},\{0,7,25\},\{8,9,22\},\{2,11,18\},\{2,9,17\}$,
$\{13,17,26\},\{6,14,22\},\{3,16,18\},\{12,14,18\},\{5,20,21\},\{10,20,23\}$,
$\{1,14,20\},\{0,21,23\},\{6,21,25\},\{5,12,25\},\{10,17,18\},\{4,19,23\}$,
$\{7,17,24\},\{1,9,13\},\{1,2,26\},\{2,10,25\},\{2,15,21\},\{1,17,21\},\{7,13,23\}$,
$\{0,2,19\},\{13,16,22\},\{0,5,24\},\{7,8,10\},\{4,9,24\},\{22,24,26\},\{3,19,20\}$, $\{9,14,19\},\{9,15,25\},\{11,21,26\},\{4,13,25\},\{0,8,16\},\{3,14,25\}$,
$\{8,15,24\},\{8,19,21\},\{3,21,22\},\{7,18,26\},\{0,6,20\},\{4,16,21\},\{8,20,25\}$,
$\{1,10,24\},\{3,10,11\},\{0,10,26\},\{18,24,25\},\{11,23,25\},\{5,9,18\}$
$\{1,3,8\},\{0,3,6\},\{3,4,5\},\{0,5,7\},\{2,5,8\},\{1,5,6\},\{6,7,8\}$
$\{0,1,2\},\{2,3,7\},\{0,4,8\},\{2,4,6\},\{1,4,7\}$
$\{6,15,26\},\{1,6,18\},\{16,24,26\},\{5,6,23\},\{5,13,17\},\{6,21,25\},\{2,5,24\}$, $\{3,10,14\},\{9,15,21\},\{2,9,13\},\{7,17,18\},\{18,19,23\},\{1,15,23\}$, $\{10,24,25\},\{0,5,16\},\{8,10,23\},\{3,6,24\},\{13,15,16\},\{1,8,16\},\{2,16,17\}$, $\{4,15,18\},\{13,19,26\},\{16,20,25\},\{9,18,22\},\{3,20,23\},\{13,23,24\}$, $\{6,20,22\},\{12,22,25\},\{7,9,26\},\{4,11,26\},\{7,12,20\},\{5,10,21\}$, $\{4,23,25\},\{0,14,26\},\{7,16,23\},\{1,11,21\},\{7,11,22\},\{4,9,20\}$, $\{14,18,25\},\{3,12,15\},\{3,16,18\},\{1,10,19\},\{19,20,21\},\{21,22,23\}$, $\{3,4,19\},\{0,11,23\},\{0,10,18\},\{6,7,10\},\{2,23,26\},\{9,14,23\},\{3,11,17\}$, $\{7,13,25\},\{5,15,25\},\{2,12,21\},\{14,16,21\},\{2,14,20\},\{1,22,24\}$, $\{5,11,18\},\{1,3,5\},\{8,15,24\},\{3,8,22\},\{2,19,22\},\{1,13,20\},\{10,20,26\}$, $\{11,19,24\},\{3,25,26\},\{8,17,25\},\{5,9,12\},\{12,13,18\},\{6,14,17\}$, $\{5,22,26\},\{4,17,21\},\{0,17,20\},\{8,9,11\},\{0,21,24\},\{7,14,24\},\{5,7,19\}$, $\{2,8,18\},\{2,10,15\},\{6,11,16\},\{0,6,12\},\{10,11,12\},\{1,17,26\},\{4,16,22\}$, $\{5,8,20\},\{4,5,14\},\{4,10,13\},\{6,9,19\},\{11,15,20\},\{18,20,24\},\{8,12,26\}$, $\{15,17,19\},\{12,17,23\},\{10,17,22\},\{7,8,21\},\{12,16,19\},\{2,11,25\}$, $\{3,13,21\},\{1,12,14\},\{8,14,19\},\{11,13,14\},\{18,21,26\},\{4,12,24\}$, $\{14,15,22\},\{0,7,15\},\{0,13,22\},\{1,9,25\},\{9,17,24\},\{6,8,13\},\{0,19,25\}$, $\{0,3,9\},\{9,10,16\}$

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