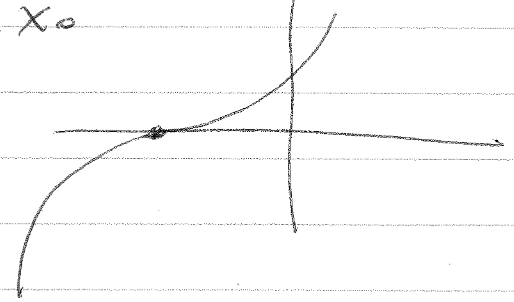
Consider $\begin{cases} y' = \sqrt{|y|} \\ y(x_0) = 0 \end{cases}$

By observation, $y(x) = 0$ is a solution

By separation of variables,

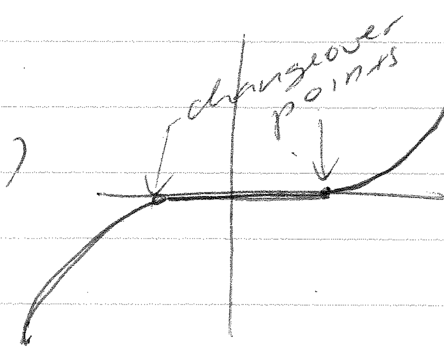
$$y(x) = \begin{cases} \dfrac{(x-x_0)^2}{4} & x \geq x_0 \\ -\dfrac{(x-x_0)^2}{4} & x < x_0 \end{cases}$$

is a solution.



Fact: I can create a Franken-Solution

$$y(x) = \begin{cases} \dfrac{(x-2)^2}{4} & x \geq 2 \\ 0 & x \in (-1, 2) \\ -\dfrac{(x+1)^2}{4} & x \leq -1 \end{cases}$$




this also solves $y' = \sqrt{|y|}$!

I can create infinitely many Franken-Solutions ... they are all legitimate Solutions

$\Rightarrow \begin{cases} y' = \sqrt{|y|} \\ y(x_0) = 0 \end{cases}$ has infinitely many solutions (just make sure $x_0$ is a changeover point