ex: $x' = x^2 + y$

$y' = x - y + a$

$y' = 0 \Rightarrow y = x + a$

$x' = 0 \Rightarrow x^2 + x + a = 0$

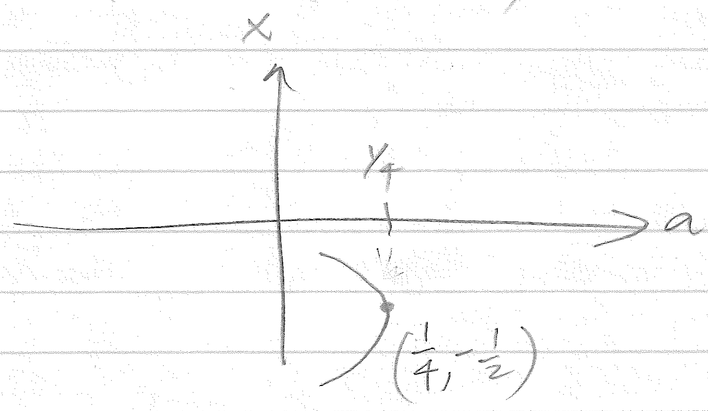
$$\Rightarrow x_{ss} = \frac{-1 \pm \sqrt{1-4a}}{2}$$

$$x_{ss} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - a}$$

So $a_0 = 1/4$, $x_0 = -1/2$ is bifurcation pt.

$a < 1/4 \Rightarrow \exists$ 2 steady states

$a > 1/4 \Rightarrow \nexists$ steady states.

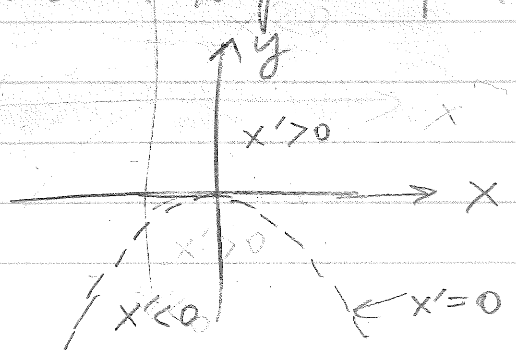


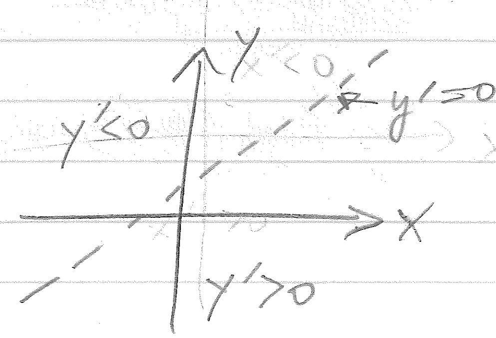
$x' > 0$

$x^2 + y > 0$

$y > -x^2$

Let's look at the phase portraits.

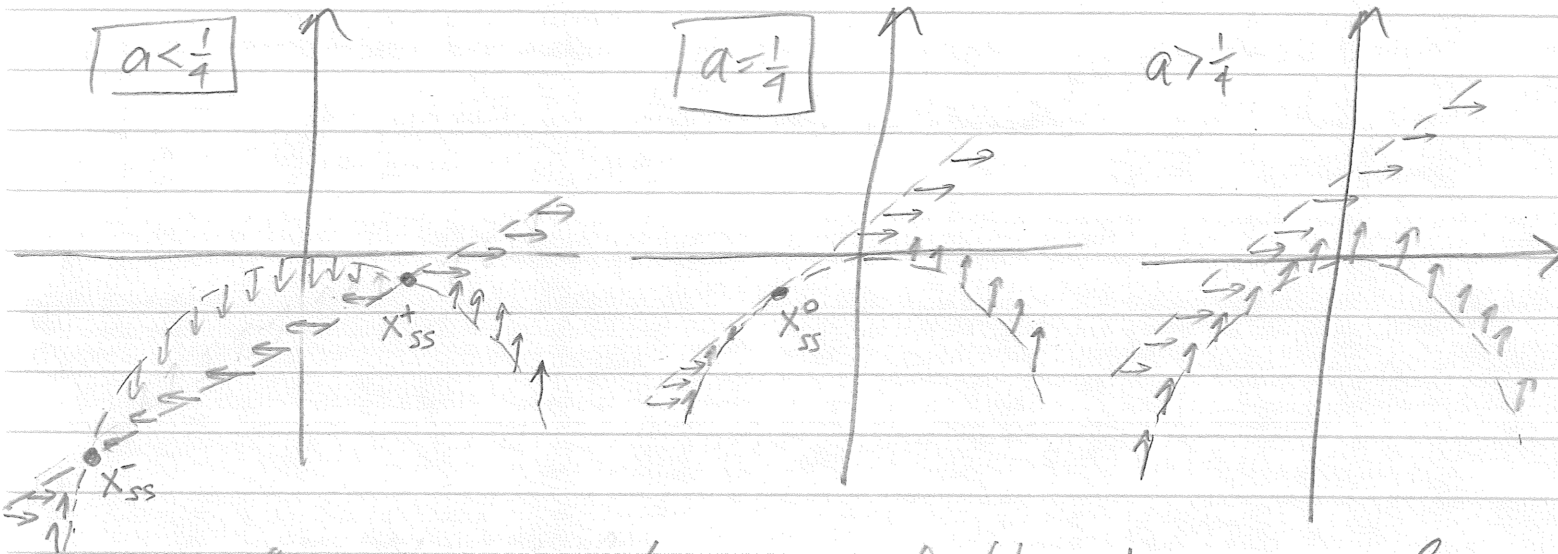




fixed points occur where the

$x'=0$ curve $y=-x^2$

and $y'=0$ curve $y=x+a$ intersect,



on these curves, the vector field is horizontal or vertical.

Do linear stability analysis about the fixed points

$$\vec{F}=\begin{pmatrix} x^2+y \\ x-y+a \end{pmatrix} \qquad DF=\begin{pmatrix} 2x & 1 \\ 1 & -1 \end{pmatrix}$$

$$X_{ss}^{-}=-\frac{1}{2}-\sqrt{\frac{1}{4}-a} \qquad DF(X_{ss}^{-})=\begin{pmatrix} -1-2\sqrt{1/4-a} & 1 \\ 1 & -1 \end{pmatrix}$$

$$\lambda_{\pm}=\frac{1}{2}\left(-2-\sqrt{1-4a}\pm\sqrt{5-4a}\right)$$ both negative if $a<1/4$

$$X_{ss}^{+}=-\frac{1}{2}+\sqrt{\frac{1}{4}-a} \qquad DF(X_{ss}^{+})=\begin{pmatrix} -1+2\sqrt{1/4-a} & 1 \\ 1 & -1 \end{pmatrix}$$

$$\lambda_{\pm}=\frac{1}{2}\left(-2+\sqrt{1-4a}\pm\sqrt{5-4a}\right)$$ one positive, one neg.

Doing the analysis of where $x' = 0$ and where $y' = 0$ was suggestive of x_{ss}^- being stable and x_{ss}^+ being unstable but without computing DF at these fixed points and finding the eigenvalues, it's just pictures.

<u>Note:</u>

x_{ss}^- has eigenvalue

$$\lambda^+ = \frac{1}{2}\left(-2 - \sqrt{1-4a} + \sqrt{5-4a}\right)$$

as $a \uparrow 1/4$, $\lambda^+ \uparrow 0$. λ^- stays negative as $a \uparrow 0$

x_{ss}^+ has eigenvalue

$$\lambda^+ = \frac{1}{2}\left(-2 + \sqrt{1-4a} + \sqrt{5-4a}\right)$$

as $a \uparrow 1/4$, $\lambda^+ \downarrow 0$. λ^- stays negative as $a \uparrow 0$