Note: I had a sign error in the V I gave in class.

$$L = \begin{pmatrix} 4 & 7 \\ 8 & -1 \end{pmatrix} \Rightarrow LL^T = \begin{pmatrix} 4 & 7 \\ 8 & -1 \end{pmatrix}\begin{pmatrix} 4 & 8 \\ 7 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 65 & 25 \\ 25 & 65 \end{pmatrix}$$

eigenvalues of LL^T? $\det\begin{pmatrix} 65-\lambda & 25 \\ 25 & 65-\lambda \end{pmatrix}$

$$= (65-\lambda)^2 - 25^2$$

$$= \lambda^2 - 130\lambda + 3600$$

$$= (\lambda-40)(\lambda-90) \qquad \lambda_1 = 40 \quad \lambda_2 = 90$$

eigenvectors?

$$\lambda_1 = 40 \qquad \begin{pmatrix} 65-40 & 25 \\ 25 & 65-40 \end{pmatrix}\vec{u}_1 = \vec{0} \Rightarrow \vec{u}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 90 \qquad \begin{pmatrix} 65-90 & 25 \\ 25 & 65-90 \end{pmatrix}\vec{u}_2 = 0 \Rightarrow \vec{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

normalize the eigenvectors $\vec{u}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\vec{u}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Compute L^TL to find V

$$\begin{pmatrix} 4 & 8 \\ 7 & -1 \end{pmatrix}\begin{pmatrix} 4 & 7 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} 80 & 20 \\ 20 & 50 \end{pmatrix}$$

eigenvalues of L^TL? Same as for LL^T because L^TL is a symmetric matrix and L is square

$$\lambda_1 = 40 \quad \begin{pmatrix} 80-40 & 20 \\ 20 & 50-40 \end{pmatrix}\vec{v}_1 = \vec{0} \Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\lambda_2 = 90 \quad \begin{pmatrix} 80-90 & 20 \\ 20 & 50-90 \end{pmatrix}\vec{v}_2 = 0 \Rightarrow \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

normalize $\vec{v}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ for $\lambda = 40$

$\vec{v}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ for $\lambda = 90$

Now construct Σ so that

$L = U\Sigma V^T$ $\quad \lambda_2 = 90$ is the larger eigenvalue so put corresp. eigenvectors in 1st col.

$$U = (\vec{u}_2 | \vec{u}_1) \quad V = (\vec{v}_2 | \vec{v}_1) \quad \Sigma = \begin{pmatrix} \sqrt{\lambda_2} & 0 \\ 0 & \sqrt{\lambda_1} \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 3\sqrt{10} & 0 \\ 0 & 2\sqrt{10} \end{pmatrix} \quad V = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$$