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1. (1 point) `Library/Rochester/setDiffEQ13Systems1stOrder/ur_de_13_3.pg`

Write the given second order equation as its equivalent system of first order equations.

$$t^2u'' - 2.5tu' + (t^2 - 7.5)u = -8\sin(3t)$$

Use v to represent the "velocity function", i.e. $v = u'(t)$.
Use v and u for the two functions, rather than $u(t)$ and $v(t)$. (The latter confuses webwork. Functions like $\sin(t)$ are ok.)

$u' =$ ______________________

$v' =$ ______________________

Now write the system using matrices:

$$\frac{d}{dt}\begin{bmatrix} \text{u} \\ \text{v} \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}\begin{bmatrix} \text{u} \\ \text{v} \end{bmatrix} + \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}.$$

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2. (1 point) `Library/Rochester/setDiffEQ13Systems1stOrder/ur_de_13_16.pg`

Multiplying the differential equation

$$\frac{df}{dt} + af(t) = g(t),$$

where a is a constant and $g(t)$ is a smooth function, by e^{at}, gives

$$e^{at}\frac{df}{dt} + e^{at}af(t) = e^{at}g(t),$$

$$\frac{d}{dt}\left(e^{at}f(t)\right) = e^{at}g(t),$$

$$e^{at}f(t) = \int e^{at}g(t)dt,$$

$$f(t) = e^{-at}\int e^{at}g(t)dt.$$

Use this to solve the initial value problem

$$\frac{dx}{dt} = \begin{bmatrix} 1 & -1 \\ 0 & -3 \end{bmatrix}x,$$

with $x(0) = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$,

i.e. find first $x_2(t)$ and then $x_1(t)$.

$x_1(t) =$ ______________________,

$x_2(t) =$ ______________________.

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3. (1 point) `Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-32.pg`

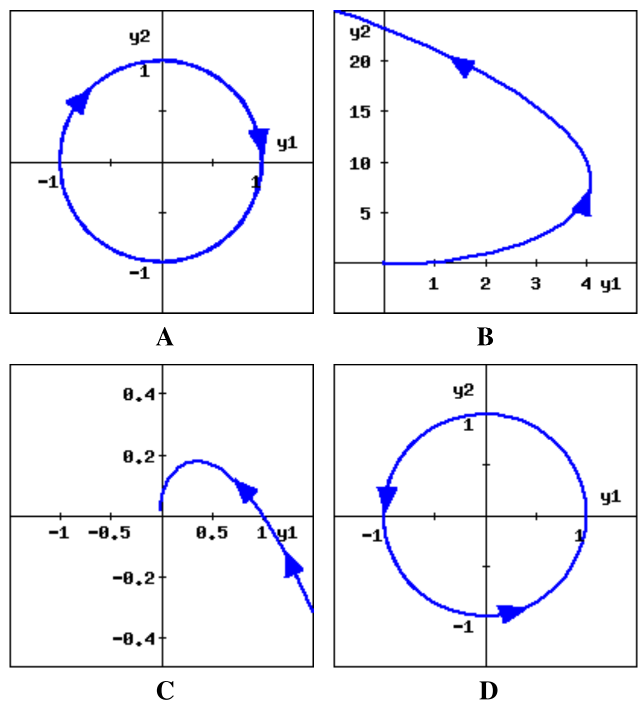
Match each initial value problem with the phase plane plot of its solution. (The arrows on the curves indicate how the solution point moves as t increases.)

[?] 1. $\vec{y}' = \begin{bmatrix} 1 & -0.5 \\ 0.5 & 1 \end{bmatrix}\vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 2. $\vec{y}' = \begin{bmatrix} -1 & -0.5 \\ 0.5 & -1 \end{bmatrix}\vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 3. $\vec{y}' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 4. $\vec{y}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$



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4. (1 point) `Library/METU-NCC/Diff_Eq/pplane-match_1.pg`

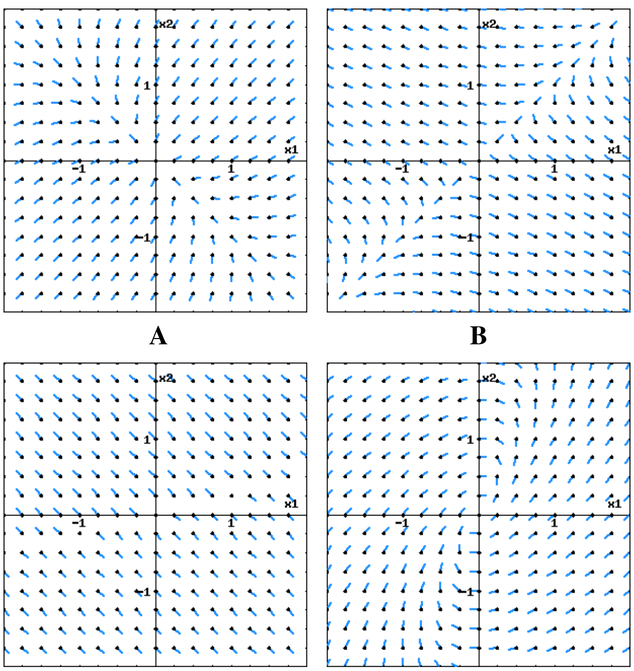
Match each linear system with one of the phase plane direction fields.
(The blue lines are the arrow shafts, and the black dots are the arrow tips.)

? 1. $\vec{x}' = \begin{bmatrix} -5 & -3 \\ -3 & -5 \end{bmatrix} \vec{x}$

? 2. $\vec{x}' = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix} \vec{x}$

? 3. $\vec{x}' = \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \vec{x}$

? 4. $\vec{x}' = \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} \vec{x}$



A B C D

Note: To solve this problem, you only need to compute eigenvalues. In fact, it is enough to just compute whether the eigenvalues are real or complex and positive or negative.

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5. **(1 point)** `Library/FortLewis/DiffEq/3-Linear-systems/05-2D-systems-vector-fields/Systems-Classification-05.pg`

(1) Find the most general real-valued solution to the linear system of differential equations $\vec{x}' = \begin{bmatrix} -5 & -16 \\ 1 & -5 \end{bmatrix} \vec{x}$.

$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = c_1 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} + c_2 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

(2) In the phase plane, this system is best described as a

- source / unstable node
- sink / stable node
- saddle
- center point / ellipses
- spiral source