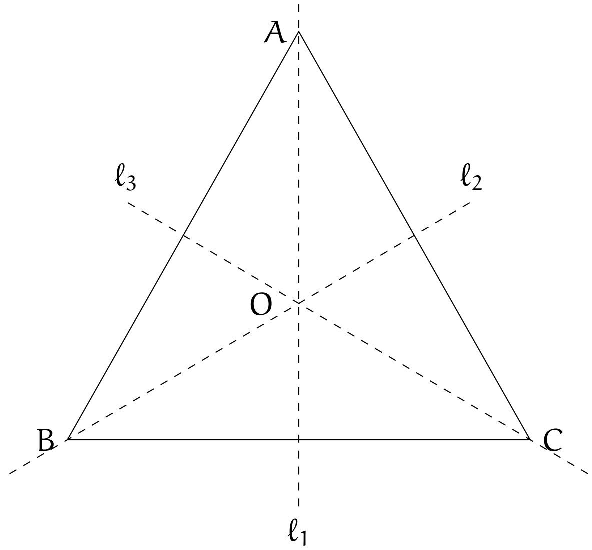
MAT301 Groups and Symmetry

Tutorial Activity: Symmetries of an Equilateral Triangle

Sep 12 and Sep 13



Let S be the set of all symmetries of the equilateral triangle above. Let ρ_1 and ρ_2 be respectively (counter-clockwise) rotations around O by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Denote the reflections about ℓ_1, ℓ_2, ℓ_3 by r_1, r_2, r_3 respectively. Let e be the identity symmetry. We have

$$S = \{e, \rho_1, \rho_2, r_1, r_2, r_3\}$$

Complete the Cayley table of S under composition of functions.

$\circ$	e	ρ_1	ρ_2	r_1	r_2	r_3
e						
ρ_1				r_3		
ρ_2						
r_1						
r_2						
r_3						

Some facts: The following facts about composition of symmetries can speed up your calculations:

(1) Composition of two rotations around O is a rotation around O. More precisely, composition of a rotation by θ around O followed by a rotation by θ' around O is a rotation by $\theta + \theta'$ around O.

(2) Let ℓ be a line passing through O. Composition of a reflection about ℓ with a rotation around O is a reflection.

(3) Let ℓ and ℓ' be two intersecting lines. Composition of reflections about ℓ and ℓ' is a rotation.

(4) Let ℓ and ℓ' be two parallel lines. Composition of reflections about ℓ and ℓ' is a translation. (This fact won't be used in calculating Cayley tables of symmetries of bounded plane figures.)

Questions:

(1) Is the binary operation $\circ$ on S commutative?

(2) What interesting features do you observe in the Cayley table? (For example, does every element of S appear in every row?)