Algebra – Problems

1. Simplify the following expressions and eliminate all negative exponents where appropriate:

(a) $\dfrac{2}{3}+\dfrac{1}{4}-\dfrac{5}{2}$

(b) $\dfrac{5}{6}+\dfrac{5}{2}\cdot\dfrac{8}{3}$

(c) $\left(\dfrac{5}{2}\right)^{-2}+\dfrac{6}{7}\cdot\dfrac{5}{2}$

(d) $\left[1-\left(\dfrac{3}{5}\right)^{2}\right]^{-\frac{1}{2}}+\left[1-\left(\dfrac{25}{9}\right)^{\frac{1}{2}}\right]^{-2}$

(e) $\left(\dfrac{7}{3}+\left(\dfrac{8}{27}\right)^{\frac{1}{3}}\right)^{3}\left(\dfrac{|3-\pi|-(\pi-12)}{|1-2|}\right)^{-\frac{3}{2}}$

(f) $\left(\dfrac{a^{\frac{1}{2}}}{b^{-\frac{2}{3}}c^{2}}\right)^{4}\left(\dfrac{ac^{-3}}{b^{\frac{1}{6}}}\right)^{-2}$

(g) $\dfrac{1000^{\frac{2}{3}}-1}{3^{2}+2\cdot 7^{0}-2}\cdot\dfrac{11^{-1}}{4^{-\frac{5}{2}}}$

2. Simplify and collect like terms:

(a) $3\sqrt{180}+4\sqrt{45}-7\sqrt{84}+\sqrt{189}-2\sqrt{245}$

(b) $\sqrt{7}\left(5\sqrt{140}-8\sqrt{120}+3\sqrt{21}\right)$

(c) $\left(2\sqrt{3}+5\sqrt{2}\right)\left(2\sqrt{3}-5\sqrt{2}\right)$

3. Simplify:

(a) $(x+1)(x-2)+(x-1)(x+1)(x-3)-2x(x^{2}-5x+8)$

(b) $\dfrac{(a-b)(2a+b)+3b(b-a)}{(a-b)^{2}-2b(b-a)}$

(c) $(x^{2}+10x+25)^{\frac{-}{1}2}\cdot(x+5)^{4}$

(d) $\dfrac{28x^{2}y^{3}z^{5}+12x^{-1}y^{5}z^{3}}{20xyz^{2}}+\dfrac{1}{3xz}$

4. Simplify the expression by rationalizing the denominator:

$$\frac{x+2}{\sqrt{x^{2}+5x+10}-x}.$$

5. Simplify the following expressions:

(a) $\dfrac{a}{a-3} - \dfrac{2}{a+4} - \dfrac{14}{a^2+a-12}$

(b) $\dfrac{x^2-x-12}{x^2+2x-3} \div \dfrac{x-2}{x^2+3x-4} + \dfrac{1}{x^2-2x}$

6. Solve the following systems of equations:

(a) $y = 2x + 4 \quad , \quad \dfrac{y}{2} - 3 = x.$

(b) $5x + 2y = 2 \quad , \quad 4x + y = 10.$

(c) $y = \dfrac{3}{2} - 5x \quad , \quad 2y + 10x = 3.$