Results from the talk on the Hasse-Minkowski Thm:

Prop. A: If $x \in V \setminus \{0\}$ is isotropic and $disc(Q) \neq 0$.

$\Rightarrow \exists$ a subspace $U \subset V$, s.th. $x \in U$ and U is a hyperbolic plane.

Cor. A1: $\exists\ x \in V \setminus \{0\}$ isotropic and $disc(Q) \neq 0 \Rightarrow Q(V) = k$.

Theorem 1. Every quad. module (V, Q) has an orthogonal basis.

Fact 1. Given two orthogonal bases $(e_i), (e_i')$ there is a finite

sequence of orthogonal bases starting with (e_i) ending with (e_i')

s.th every two consecutive ones are contiguous.

Prop. B: g, h nondegen. of *rank* ≥ 1, $f = g \dot{+} (-h) \Rightarrow$ TFAE:

(a) f represents 0, i.e. $\exists\ x \in V \setminus \{0\}$ s.th. $f(x) = 0$.

(b) $\exists\ a \in k^*$ represented by both g and h.

(c) $\exists\ a \in k^*$ such that $g \dot{+} (-aZ^2)$ and $h \dot{+} (-aZ^2)$ represent 0.

Theorem 3. $\epsilon(\mathbf{e})$ does not depend on the choice of $\mathbf{e}$.

Fact 2. Forms f, g are equivalent iff $rank(f) = rank(g)$,

$disc(f) = disc(g)$, $\epsilon(f) = \epsilon(g)$. (To be believed!)

Fact 2. Hilbert's Theorem: $a, b \in \mathbb{Q}^* \Rightarrow (a, b)_v = 1$ for all

but finitely many v and $\displaystyle\prod_{v \in \mathcal{P}} (a, b)_v = 1$. (Sadly, to be believed!)

Hasse-Minkowski Theorem:

$$0 \in Im(f) \iff 0 \in Im(f_v) \; \forall \; v \in \mathcal{P}.$$

Fact 3. Let $b_1, b_2, b_3, b_4 \in \mathbb{Q}^*$ and $\{\epsilon_{i,v}\}_{i \in I, v \in V} \in \{\pm 1\}$ with

(1) all but finitely many $\epsilon_{i,v} = 1$;

(2) for all $i \in I$ we have $\prod_v \epsilon_{i,v} = 1$;

(3) for all $v \in \mathcal{P}$ $\exists$ $a_v \in \mathbb{Q}_v^*$ s.th. $(b_i, a_v)_v = \epsilon_{i,v}$ $\forall v \in \mathcal{P}$

then there exists $a \in \mathbb{Q}^*$ s.th $(b_i, a)_v = \epsilon_{i,v}$ for all $i \in I, v \in V$.

Fact 4.: $S \subset \mathcal{P}$, $|S| < \infty \Rightarrow$ image of $\mathbb{Q}$ in $\prod_{v \in S} \mathbb{Q}_v$ is dense.

Cor. B1: $a \in \mathbb{Q}$. Then f represents a in $\mathbb{Q}$ iff it does in each $\mathbb{Q}_v$.

Cor. B2: (Meyer). A quadratic for of rank ≥ 5 represents 0 in $\mathbb{Q}$ iff it does so in $\mathbb{R}$. (In such case 0 is represented in all $\mathbb{Q}_v$.)