APM 351: Differential Equations in Mathematical Physics
Announcement of Second Midterm Test

When and where?

Wednesday, January 25, 5-7pm, EX 310 (Exam Centre) Closed books, closed notes.

What is covered?

The test will cover Chapter 5-7 of Strauss, Assignments 8-12, and selected topics from the earlier part of the course. Specifically:

- *Hilbert spaces.* Inner product and norm. Orthogonality. Bessel's inequality, Parseval's identity and completeness. Orthonormal bases. The space L^2 of square integrable periodic functions. Mean square convergence vs. uniform convergence.
- *Fourier series.* Sine, cosine, and complex exponential series. How to compute the Fourier coefficients of a given function. Symmetric boundary conditions and self-adjointness of $-\partial_x^2$.
- *Harmonic functions.* Physical motivation. Laplace's equation in Cartesian and polar coordinates. Relationship with holomorphic function. Weak and strong maximum principle. Mean value property. Poisson's formula for the disk. Half-space and ball.
- *Laplace's equation and Poisson's equation.* Dirichlet and Neumann problems. Uniqueness (for bounded domains) and non-uniqueness (on the entire space). Energy methods. The fundamental solution of the Laplacian on $\mathbb{R}^2$ and $\mathbb{R}^3$. Green's identities and Green's functions.

Older topics:

- *The wave equation on the real line.* The general form of the solution; D'Alembert's formula. Causality and energy; finite speed of propagation, domain of dependence, domain of influence.
- *The heat equation on the real line.* The fundamental solution. Maximum principle and energy methods. Infinite speed of propagation. Uniquenss (on bounded domains) and non-uniqueness (on the real line).
- *Reflections and sources.* Solving boundary-value problems by even and odd reflection. Inhomogeneous equations; the Duhamel principle.
- *Separation of variables.* Cartesian and polar coordinates. Boundary-value problems for the wave and heat equation. Dirichlet, Neumann, Robin, and periodic boundary conditions. Eigenvalues and eigenfunctions. Self-adjointness and its implication for eigenvalues and eigenfunctions. Green's identity.