MAT 351: Partial Differential Equations
Test 1, November 9 2016

(Three problems; 20 points each.)

1. (a) Is the PDE
$$\sin y u_x + u_y + 2u = 0$$
linear or nonlinear? What is its order?

   (b) Write down the characteristic equations and sketch a few characteristics in the x-y-plane.

   (c) Solve this PDE with initial values $u(x, 0) = g(x)$, where g is a given function. Does your solution exist on the entire plane $\mathbb{R}^2$? Is it unique? Why?

   (d) Suppose, instead, we try to solve the same PDE with initial values $u(0, y) = h(y)$. What goes wrong?

2. (a) For the **wave equation** on the real line: Write down the initial-value problem. Briefly explain the terms *finite speed of propagation, conservation of energy*.

   (b) Write down **Burger's equation**. State the *Rankine-Hugoniot shock condition* and *Lax's entropy condition*. Use a sketch and one or two sentences to explain how these conditions determine the motion of shocks.

3. (a) Write down the fundamental solution of the heat equation $u_t = ku_{kk}$ on the real line.

   (b) Use the method of reflections to solve the heat equation on the half-line
$$u_t = ku_{xx}\,, \quad (x > 0, t > 0)$$
with Dirichlet boundary condition $u(0, t) = 0$ for $t > 0$, and initial values $u(x, 0) = \phi(x)$.

   (c) Suppose the initial values in (b) are given by
$$\phi(x) = \begin{cases} \sin^2 x\,, & \text{if } 0 < x < \pi\,, \\ 0 & \text{otherwise}\,. \end{cases}$$
Prove that the solution is strictly positive, $u(x, t) > 0$ for all $x > 0, t > 0$.