

Dror Bar-Natan — Curriculum Vitae

drorbn@math.toronto.edu

$\omega\varepsilon\beta$:= <https://www.math.toronto.edu/~drorbn>

See a fully linked version at [\$\omega\varepsilon\beta\$ /Profile/BarNatanCV.pdf](https://www.math.toronto.edu/~drorbn/Profile/BarNatanCV.pdf)

Last updated: Apr. 20, 2025

- **1982–1984:** B.Sc. in mathematics, Tel Aviv University, *summa cum laude*.
- **1984–1987:** Military service (taught high school level mathematics).
- **1987–1991:** Ph.D. in mathematics, Princeton University. Title: *Perturbative aspects of the Chern-Simons topological quantum field theory*. Advisor: Professor Edward Witten.
- **1991–1995:** Benjamin Peirce Assistant Professor at Harvard University.
- **1995–1997:** Senior Lecturer of Mathematics at the Hebrew University, Jerusalem.
- **1997–2004:** Associate Professor of Mathematics at the Hebrew University, Jerusalem.
- **1999-2000:** On sabbatical in Berkeley; fall semester as a “Visiting Miller Professor” at the University of California, spring semester at MSRI.
- **2002-2006:** Associate Professor of Mathematics at the University of Toronto.
- **From July 2006:** Professor of Mathematics at the University of Toronto.

Publications in peer reviewed journals and conference proceedings (the symbol “*” indicates the most significant): (See also [\$\omega\varepsilon\beta\$ /LOP.html](https://www.math.toronto.edu/~drorbn/LOP.html))

1. *Two Examples in Non-Commutative Probability*, Foundations of Physics **19** (1989) 97–104.
2. *Perturbative Chern-Simons Theory*, Journal of Knot Theory and its Ramifications, **4-4** (1995) 503–548.
3. With E. Witten, *Perturbative Expansion of Chern-Simons Theory with Non-Compact Gauge Group*, Commun. Math. Phys. **141** (1991) 423–440.
4. * *On the Vassiliev Knot Invariants*, Topology **34** (1995) 423–472. (Reported at the prestigious Séminaire Bourbaki. See P. Vogel, *Invariants de Vassiliev des nœuds [d’après D. Bar-Natan, M. Kontsevich et V. A. Vassiliev]*, Séminaire Bourbaki **761** (1993) 1–17 & Asterisque **216** (1993) 213–232).
5. *Vassiliev Homotopy String Link Invariants*, Journal of Knot Theory and its Ramifications **4** (1995) 13–32.
6. * *Non-Associative Tangles*, in *Geometric topology* (proceedings of the Georgia international topology conference), (W. H. Kazez, ed.), 139–183, Amer. Math. Soc. and International Press, Providence, 1997.
7. * With S. Garoufalidis, *On the Melvin-Morton-Rozansky Conjecture*, Inventiones Mathematicae **125** (1996) 103–133.
8. *Vassiliev and Quantum Invariants of Braids*, Proceedings of Symposia in Applied Mathematics **51** (1996) 129–144, *The interface of knots and physics*, (L. H. Kauffman, ed.), American Mathematical Society.
9. *Polynomial Invariants are Polynomial*, Mathematical Research Letters **2** (1995) 239–246.
10. With J. Fulman and L. Kauffman, *An Elementary Proof that All Seifert Surfaces of a Link are Tube-Equivalent*, Journal of Knot Theory and its Ramifications **7-7** (1998) 873–879.

11. * *Lie Algebras and the Four Color Theorem*, *Combinatorica* **17-1** (1997) 43–52.
12. With A. Stoimenow, *The Fundamental Theorem of Vassiliev Invariants*, in *Geometry and Physics*, (J. E. Andersen, J. Dupont, H. Pedersen, and A. Swann, eds.), lecture notes in pure and applied mathematics **184**, Marcel Dekker, New-York 1997, pp. 101–134.
13. * *On Associators and the Grothendieck-Teichmüller Group*, *Selecta Mathematica (New Series)* **4** (1998) 183–212.
14. * With S. Garoufalidis, L. Rozansky, and D. P. Thurston, *Wheels, Wheeling, and the Kontsevich Integral of the Unknot*, *Israel Journal of Mathematics* **119** (2000) 217–237.
15. * With S. Garoufalidis, L. Rozansky, and D. P. Thurston, *The Århus Integral of Rational Homology 3-Spheres I: A Highly Non Trivial Flat Connection on S^3* , *Selecta Mathematica, New Series* **8** (2002) 315–339.
16. With S. Garoufalidis, L. Rozansky, and D. P. Thurston, *The Århus Integral of Rational Homology 3-Spheres II: Invariance and Universality*, *Selecta Mathematica, New Series* **8** (2002) 341–371.
17. * With B. McKay, M. Bar-Hillel and G. Kalai, *Solving the Bible Code Puzzle*, *Statistical Science* **14-2** (1999) 150–173.
18. With R. Lawrence, *A Rational Surgery Formula for the LMO Invariant*, *Israel Journal of Mathematics* **140** (2004) 29–60.
19. *On Khovanov’s Categorification of the Jones Polynomial*, $\omega\epsilon\beta$ /papers/Categorification, *Algebraic and Geometric Topology* **2-16** (2002) 337–370.
20. *Bracelets and the Goussarov Filtration of the Space of Knots, Invariants of Knots and 3-Manifolds (Kyoto 2001)*, *Geometry and Topology Monographs* **4** 1–12.
21. * With T. Q. T. Le and D. P. Thurston, *Two Applications of Elementary Knot Theory to Lie Algebras and Vassiliev Invariants*, *Geometry and Topology* **7-1** (2003) 1–31.
22. With S. Garoufalidis, L. Rozansky, and D. P. Thurston, *The Århus Integral of Rational Homology 3-Spheres III: The Relation with the Le-Murakami-Ohtsuki Invariant*, *Selecta Mathematica, New Series* **10** (2004) 305–324.
23. *Khovanov Homology for Knots and Links with up to 11 Crossings*, Proceedings of the NATO Advanced Research Workshop on New Techniques in Topological Quantum Field Theory, Calgary Summer 2001, about 74 pp.
24. *Finite Type Invariants*, *Encyclopedia of Mathematical Physics* (eds. J.-P. Francoise, G. L. Naber and Tsou S. T.) vol. **2** pp. 340, Elsevier, Oxford 2006.
25. * *Khovanov’s Homology for Tangles and Cobordisms*, *Geometry and Topology* **9-33** (2005) 1443–1499.
26. * *Fast Khovanov Homology Computations*, *Journal of Knot Theory and its Ramifications* **16-3** (2007) 243–255.
27. With S. Morrison, *The Karoubi Envelope and Lee’s Degeneration of Khovanov Homology*, *Algebraic and Geometric Topology* **6** (2006) 1459–1469.
28. With I. Halacheva, L. Leung and F. Roukema, *Some Dimensions of Spaces of Finite Type Invariants of Virtual Knots*, *Experimental Mathematics* **20-3** (2011) 282–287.
29. With Z. Dancso, *Pentagon and Hexagon Equations Following Furusho*, *Proceedings of the American Mathematical Society* **140-4** (2012) 1243–1250.
30. With Z. Dancso, *Homomorphic Expansions for Knotted Trivalent Graphs*, *Journal of Knot Theory and Its Ramifications* **22-1** (2013), 33 pp.
31. *Review of a Book by Chmutov, Duzhin, and Mostovoy*, *Bulletin of the American Mathematical Society* **50** (2013) 685–690.

32. With S. Selmani, [Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial](#), Journal of Knot Theory and Its Ramifications **22-10** (2013) (14 pp), [ωεβ/papers/MetaMonoids.html](#).
33. With H. Burgos-Soto, [Khovanov Homology for Alternating Tangles](#), Journal of Knot Theory and its Ramifications **23-2** (2014) (18 pp), [arXiv:1305.1695](#).
34. * [Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant](#), Acta Mathematica Vietnamica **40-2** (2015) 271–329, [ωεβ/papers/KBH/](#).
35. With Huan Vo, [Proof of a Conjecture of Kulakova et al. Related to the \$sl_2\$ Weight System](#), European Journal of Combinatorics **45** (2015) 65–70, [arXiv:1401.0754](#).
36. With Z. Dancso, [Finite Type Invariants of W-Knotted Objects I: W-Knots and the Alexander Polynomial](#), Algebraic and Geometric Topology **16-2** (2016) 1063–1133, [arXiv:1405.1956](#).
37. * With Z. Dancso, [Finite Type Invariants of W-Knotted Objects II: Tangles, Foams, and the Kashiwara-Vergne Problem](#), Mathematische Annalen **367** (2017) 1517–1586, [arXiv:1405.1955](#).
38. [A Note on the Unitarity Property of the Gassner Invariant](#), Bulletin of Chelyabinsk State University (Mathematics, Mechanics, Informatics) **3-358-17** (2015) 22–25, [arXiv:1406.7632](#).
39. [On Raoul Bott’s “On Invariants of Manifold”](#), in Bott’s collected works **5**, 2pp.
40. With R. van der Veen, [A Polynomial Time Knot Polynomial](#) Proc. Amer. Math. Soc. **147** (2019) 377–397, [arXiv:1708.04853](#).
41. With Z. Dancso and N. Scherich, [Ribbon 2-Knots, \$1 + 1 = 2\$, and Duflo’s Theorem for Arbitrary Lie Algebras](#), Algebraic and Geometric Topology **20** (2020) 3733–3760, [arXiv:1811.08558](#).
42. With R. van der Veen, [An Unexpected Cyclic Symmetry of \$Iu_n\$](#) , Abh. Math. Semin. Univ. Hamburg (2023), [arXiv:2002.00697](#) (6 pp.).
43. With Z. Dancso and R. van der Veen, [Over then Under Tangles](#), Journal of Knot Theory and its Ramifications **32-8** (2023) (40 pp), [arXiv:2007.09828](#).
44. With I. Bar-Natan, I. Halacheva, and N. Scherich, [Yarn Ball Knots and Faster Computations](#), Journal of Applied and Computational Topology **8** (2024) 175–192, [arXiv:2108.10923](#).
45. * With R. van der Veen, [A Perturbed-Alexander Invariant](#), Quantum Topology **15** (2024) 449–472, [arXiv:2206.12298](#), 24pp.
46. With I. Bar-Natan, I. Halacheva, and N. Scherich, [Computing Finite Type Invariants Efficiently](#), to appear in Proc. Amer. Math. Soc., [arXiv:2408.15942](#).

Publications (other):

47. [Random Dot Stereograms](#), The Mathematica Journal **1-3** (1991) 69–75.
48. [Weights of Feynman Diagrams and the Vassiliev Knot Invariants](#), February 1991 preprint, 22 pp.
49. [Some Computations Related to Vassiliev Invariants](#), [ωεβ/papers/table.dvi](#), July 1994, 18 pp.
50. With B. McKay, [Equidistant Letter Sequences in Tolstoy’s “War and Peace”](#), [ωεβ/Codes/WNP](#), September 1997, 12 pp.
51. With B. McKay and S. Sternberg, [On the Witztum-Rips-Rosenberg Sample of Nations](#), [ωεβ/Codes/Nations](#), March 1998, 52 pp.
52. With M. Bar-Hillel and B. McKay, [The Torah Codes: Puzzle and Solution](#), Chance **11-2** (1998) 13–19.
53. [Algebraic Knot Theory — A Call for Action](#), [ωεβ/Paperlets.html](#), 2006 webication.
54. [The Existence of the Exponential Function](#), [ωεβ/Paperlets.html](#), 2006 webication (parts by O. A. Camarena).
55. With Z. Dancso, [Finite Type Invariants of W-Knotted Objects: From Alexander to Kashiwara and Vergne](#), [ωεβ/papers/WKO/](#), completed, 100 pp.

56. With Z. Dancso, *Finite Type Invariants of W-Knotted Objects III: The Double Tree Construction*, in preparation.
57. *Finite Type Invariants of W-Knotted Objects IV: Some Computations*, completed, awaiting submission.
58. *Expansions and Quadraticity for Groups*, in preparation.
59. With R. van der Veen, *Everything Around sl_{2+} is DoPeGDO. Hooray!*, in preparation.
60. With R. van der Veen, *Perturbed Gaussian Generating Functions for Universal Knot Invariants*, arXiv:2109.02057, 61 pp.
61. Appendix to *Emergent Version of Drinfeld's Associator Equations* by Yusuke Kuno, 2 pp., arXiv:2504.02549.
62. With Z. Dancso, T. Hogan, J. Liu, and N. Scherich, *Goldman-Turaev Formality from the Kontsevich Integral*, in preparation.
63. With R. van der Veen, *A Very Fast, Very Strong, Topologically Meaningful and Fun Knot Invariant*, in preparation.

Invited Addresses: (See also [ωεβ/Talks](#))

1. *Perturbative Chern-Simons Theory*, Austin, Texas September 1990.
2. *Chern-Simons Theory with "Formal" Gauge Group*, MSRI, Berkeley, June 1991 and again after a year of progress, Kiev, June 1992.
3. *Vassiliev Invariants and Lie Algebras*, Lebedev Physical Institute, Moscow, June 1992.
4. *On the Vassiliev Knot Invariants*, series of three lectures, given at the Newton Institute, November 1992 and in UC San Diego, January 1993.
5. *Computing Vassiliev Invariants*, Georgia International Topology Conference, and, Workshop on Conformal Field Theory, Operator Algebras and Low-Dimensional Topology, Warwick, August 1993.
6. *Vassiliev and Quantum Invariants of Braids*, AMS short course, San Francisco, January 1995.
7. *Pure Braids, the Grothendieck-Teichmüller Groups and Associators*, Workshop of Low Dimensional Topology, the Fields Institute, Waterloo, April 1995
8. *Lie algebras and the Four Color Theorem*, conference on Operads and Homotopical Algebra, Luminy, June 1995.
9. *The Fundamental Theorem of Vassiliev Invariants*, series of four lectures in a summer school on Geometry and Physics, Odense, July 1995.
10. *On the Melvin-Morton-Rozansky Conjecture*, conference on Geometry and Physics, Århus, July 1995.
11. *Physics in Lillienblum Street*, Hebrew University Colloquium, November 1995.
12. *Finite-Type Invariants of Knots and 3-Manifolds*, series of 12 lectures in the Mathematics Research at the Korea Advanced Institute of Science and Technology (KAIST), August 1996.
13. *Wheels, Wheeling, and the Kontsevich Integral of the Unknot*, Mathematical Sciences Research Institute, Berkeley, January 1997.
14. *The Århus Integral of Rational Homology 3-Spheres*, Israel Mathematical Union plenary address, May 1997.
15. *Integration on Spaces of Diagrams*, Delphi, August 1998.
16. *1+1=2, The Hopf Link, and the Harish-Chandra-Duflo Isomorphism*, Århus, November 1998.
17. *From Astrology to Topology via Feynman Diagrams and Lie Algebras*, series of three lectures in a conference in Srni, the Czech Republic, January 1999. See [ωεβ/Talks/Srni-9901](#).

18. *On Links, Functions, Integrals and 3-Manifold Invariants*, University of Maryland, October 1999. See [ωεβ/Talks/UMD-991029](#).
19. *Embedded Trivalent Graphs and an Infant Conjecture*, Berkeley, February 2000. See [ωεβ/Talks/UCB-000215](#).
20. *From Stonehenge to Drinfel'd skipping all the details*, Berkeley, April 2000. See [ωεβ/Talks/UCB-000420](#).
21. *Knot Invariants, Associators and a Strange Breed of Planar Algebras*, The Fields Institute, January 2001. See [ωεβ/Talks/Fields-010111](#).
22. *A Categorification Quickie*, Calgary, August 2001. See [ωεβ/Talks/Calgary-010824](#).
23. *Bracelets and the Goussarov Filtration on the Space of Knots*, Kyoto, September 2001. See [ωεβ/Talks/Kyoto-0109](#).
24. *The Unreasonable Affinity of Knot Theory and the Algebraic Sciences*, Toronto, February 2002. See [ωεβ/Talks/Affinity/](#).
25. *Khovanov's Homology for Tangles and Cobordisms*, North Carolina, February 2003. See [ωεβ/Talks/Categorification/](#).
26. *Probability: Fact, Fiction and Quantum*, University of Western Ontario, February 2004. See [ωεβ/Talks/QuantumProbability/UWO-040213.html](#)
27. *New and Newer Knot Invariants*, 3 lectures in Oporto (Portugal), July 2004. See [ωεβ/Talks/Oporto-0407/](#)
28. *I've Computed $Kh(T(9,5))$ and I'm Happy*, Plenary lecture in Knots in Washington XX, George Washington University, February 2005. See [ωεβ/Talks/GWU-050213/](#)
29. *Local Khovanov Homology — Computations and Mutations*, Quantum Topology — Contemporary Issues and Perspectives, Utah, June 2005, See [ωεβ/Talks/Utah-0506/](#)
30. *Algebraic Knot Theory*, Jerusalem, December 2006. See [ωεβ/Talks/HUJI-061228/](#)
31. *Following Lin: Expansions for Groups*, Hanoi, August 2007. See [ωεβ/Talks/Hanoi-0708/](#)
32. *A Very Non-Planar Very Planar Algebra*, The Fields Institute, September 2007. See [ωεβ/Talks/Fields-0709/](#)
33. *Local Khovanov Homology*, Zurich Colloquium in Mathematics, May 2008.
34. *Projectivization, W-Knots, Kashiwara-Vergne and Alekseev-Torossian*, MSRI, August 2008.
35. *The Penultimate Alexander Invariant*, Sandbjerg (Denmark), October 2008.
36. *(u, v, and w knots) x (topology, combinatorics, low algebra, and high algebra)*, Twenty-Seventh Annual Friends of Mathematics Lecture, Kansas State University, April 2009.
37. *Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots*, Bonn, August 2009.
38. *u, v, and w-Knots: Topology, Combinatorics and Low and High Algebra*, Courant Lecture Series at Göttingen, April 2010
39. *I understand Drinfel'd and Alekseev-Torossian, I don't understand Etingof-Kazhdan yet, and I'm clueless about Kontsevich*, 3 lectures in Montpellier, June 2010.
40. *From the $ax+b$ Lie Algebra to the Alexander Polynomial and Beyond*, and *18 Conjectures*, two talks in Chicago, September 2010.
41. *Cosmic Coincidences and Several Other Stories*, University of Tennessee Colloquium, March 2011.
42. *Facts and Dreams About v-Knots and Etingof-Kazhdan*, at Swiss Knots 2011, May 2011.
43. *Expansions: A Loosely Tied Traverse from Feynman Diagrams to Quantum Algebra*, 6 lectures in a summer school on "Geometric, Algebraic, and Topological Methods for Quantum Field Theory", Villa de Leyva, Colombia, July 2011.

44. *Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial*, plenary lecture in the 2012 summer meeting in Regina, June 2012.
45. *Caen Workshop on v- and w-Knotted Objects*, about 18 hours of lecturing in Caen, June 2012 (including colloquium).
46. *A Quick Introduction to Khovanov Homology*, in “New Perspectives in Topological Field Theories”, Hamburg August 2012.
47. *Braids and the Grothendieck-Teichmuller Group*, the Newton Institute, January 2013.
48. *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, University of Oxford, January 2013.
49. *Trees and Wheels and Balloons and Hoops and Why I Care*, University of Toronto colloquium, March 2013.
50. *(u, v, and w knots) x (topology, combinatorics, low algebra, and high algebra)*, Master Class in Aarhus (about 35 hours lecturing, May-June 2013).
51. *A Quick Introduction to Khovanov Homology I-II and Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial*, 3 talks in Montreal, June 2013.
52. *Finite Type Invariants of Ribbon Knotted Balloons and Hoops*, Geneva, October 2013.
53. *Informal Talks on the Topology, Combinatorics, and Low and High Algebra of w-Knots*, 6 hours in Zurich, October, 2013.
54. *The Kashiwara-Vergne Problem and Topology*, Bern Colloquium, November 2013.
55. *A Partial Reduction of BF Theory to Combinatorics*, Modern Trends in Topological Quantum Field Theory, Vienna, February 2014.
56. *Some very good formulas for the Alexander polynomial*, Algebraic Structures in Low-Dimensional Topology, Oberwolfach, May 2014.
57. *Finite Type Invariants of Doodles*, “Legacy of Vladimir Arnold” Conference, The Fields Institute, November 2014.
58. *Tangles, Wheels, Balloons*, CMS Winter Meeting, Hamilton, December 2014.
59. *When does a group have a Taylor expansion?*, AMS Spring Eastern Sectional Meeting, Georgetown University, March 2015.
60. *Expansions*, 5 Chaire de la Vallée-Poussin talks in Louvain-la-Neuve, Belgium, June 2015.
61. *Polynomial Time Knot Polynomials*, “GRT, MZVs and associators” conference, Les Diablerets, Switzerland, August 2015 (2 hours).
62. *Crossing the Crossings*, “Knots and Representation Theory” seminar, Moscow (by web), November 2015.
63. *Gauss-Gassner Invariants*, “Knots in the Triangle” conference, North Carolina State University, April 2016.
64. *The Brute and the Hidden Paradise*, “GRT, MZVs and associators” conference, Les Diablerets, Switzerland, August 2016 (4 hours).
65. *A Poly-Time Knot Polynomial Via Solvable Approximation*, Indiana University, November 2016 (2 hours).
66. *On Elves and Invariants*, Plenary lecture in Knots in Washington XLIII, George Washington University, December 2016.
67. *What else can you do with solvable approximations?* McGill University HEP Seminar, February 2017.
68. *The Dogma is Wrong*, in “Lie Groups in Mathematics and Physics”, Les Diablerets, August 2017.

69. *Solvable Approximations of the Quantum sl_2 Portfolio*, 4 talks in a 5-day meeting on my work with van der Veen, Matemale, April 2018.
70. *Computation without Representation*, 2 talks in “Poisson geometry of moduli spaces, associators and quantum field theory”, Simons Center for Geometry and Physics, Stony Brook, May 2018.
71. *Everything around sl_{2+}^5 is DoPeGDO. So what?*, in “Quantum Topology and Hyperbolic Geometry”, Da Nang, Vietnam, May 2019.
72. *Algebraic Knot Theory*, University of Sydney Topology Seminar, September 2019.
73. *Some Feynman Diagrams in Algebra*, UCLA Topology Seminar, November 2019.
74. *Geography vs. Identity* and *Chord Diagrams, Knots, and Lie Algebras*, Talks at the CMS Winter 2019 Meeting, December 2019.
75. *Over then Under Tangles*, Trends in Low-Dimensional Topology, online, May 2020.
76. *The Alexander Polynomial is a Quantum Invariant in a Different Way*, Learning Seminar on Categorification, online, June 2020.
77. *I Still Don't Understand the Alexander Polynomial*, Bauman Moscow State Technical University Mathematical Colloquium, online, April 2021.
78. *Yarn-Ball Knots*, [K-OS] seminar, online, January 2021.
79. *Kashaev's Signature Conjecture*, CMS Winter 2021 Meeting, online, December 2021.
80. *Tangles in a Pole Dance Studio: A Reading of Massuyeau, Alekseev, and Naef*, Algebra, Topology and the Grothendieck-Teichmüller group, Les Diablerets, September 2022.
81. *Computing the Zombian of an Unfinished Columbarium*, 2023 CMS Summer Meeting, Ottawa, June 2023.
82. *Cars, Interchanges, Traffic Counters, and Pretty Darned Good Knot Invariants*, 70th Topology Symposium, Nara August 2023.
83. *Rooting the BKT for FTI*, University of Tokyo – Okinawa Institute of Science and Technology Joint Symposium of Knot theory, Tokyo September 2023.
84. *Shifted Partial Quadratics, their Pushforwards, and Signature Invariants for Tangles*, USC Topology Seminar, February 2024.
85. *Knot Invariants from Finite Dimensional Integration*, International Congress of Basic Science, Beijing, July 2024.
86. *Secondary Operations, Emergent Knots, and the Goldman-Turaev Lie Algebra*, Grothendieck-Teichmüller, Kashiwara-Vergne and MZVs, the University of Geneva, August 2024.
87. *The Strongest Genuinely Computable Knot Invariant in 2024*, University of Toronto Colloquium, October 2024.
88. *A Seifert Dream*, $N + 12$ th Southern California Topology Colloquium, Pitzer College, March 2025.

Other lectures in Trieste (summer 1990, [May 2009](#)), Technion (June 1991, January 1994, May 1997), Tel Aviv University (July 1991, April 1997), the Weizmann Institute (July 1992, January 1994), Columbia University (October 1991, March 2003, [February 2007](#), [December 2019](#)), Athens Georgia (August 1992), Albany (October 1992), Dartmouth (April 1993), Ann Arbor (March 1994), Berkeley (April 1994, [September 1999](#)), Hartford (March 1995), Bern (May 1995, April 1999), Marseilles (June 1995), Odense (July 1995), Århus (July, August 1995, [June 2007](#), [July 2015](#)), Haifa (May 1996, February 1998, [November 2001](#)), Bonn (July 1997, [May 2018](#), [January 2020](#), 2 talks), Princeton (February 1998), San Diego (February 1998, [January 2000](#)), Microsoft (February 1998, [April 2004](#)), Ben Gurion University ([November 1998](#), [January 2013](#)), Delphi (August 1998), Sydney (September, October 1998, [August 2017](#), [October 2019](#), (2), (3)), Canberra (October 1998, [September 2019](#)), Grenoble (April 1999, [November 2013](#), 2 talks), Luminy (May 1995, June 1999, [April 2010](#)), John Hopkins ([October 1999](#), [March 2003](#)), Georgia Tech ([December 1999](#), [October 2005](#)),

Caltech (February 2000), Riverside (April 2000), Lehigh (June 2000), CUNY (December 2000), MSRI (December 2000), Siegen (January 2001), Davis (August 2001), Calgary (August 2001), Kyoto (September 2001, May 2007, July 2023), Bar Ilan University (March 2002), Université du Québec à Montréal (September 2002, October 2005), McMaster University (October 2002, January 2014, September 2018, January 2024), Boston University (November 2002), North Carolina (February 2003, October 2016), Harvard (April 2003, October 2004), Cornell (May 2003), Potsdam NY (June 2003), Warsaw (July 2003), UIC (September 2003), Wayne State (November 2003), Banff (November 2003, April 2025), Queen's (January 2004), UWO (February 2004, February 2010), Michigan State (February 2004), Rochester (April 2004), GWU (May 2004, March 2013, December 2024), Buffalo (October 2004, March 2007, March 2012, September 2017), York (November 2004), UIUC (March 2005), LSU (April 2005), Amsterdam (April 2005), Oberwolfach (June 2005, May 2008), Iowa (January 2006, March 2016), Istanbul (June 2006), Uppsala (September 2006), Tokyo (May 2007, August 2023×2 + August 2023×2), Tianjin (July 2007), Brown (November 2007), Geneva (May 2008, May 2011, September 2013, October 2013, June 2022, December 2023, August 2024), Copenhagen (October 2009), Northeastern (October 2009, February 2016), PSU (February 2009), Bogota (February 2009), Paris (June 2009), Oregon (August 2011), Strasbourg (September 2011), Binghamton (March 2012), Toronto (January 2011, September 2016, November 2018, November 2020, December 2022), Hamburg (August 2012), Newton Institute (January 2013), Imperial College (January 2013), Sheffield (February 2013), Chicago (March 2013), Nha Trang (May 2013), Singapore (May 2013), Zurich (September 2013), Lausanne (November 2013), Magnitogorsk (July 2014), Carnegie Mellon (April 2014), Qinhuangdao (July 2015), Loyola (October 2015), Leiden (January 2016), Olympia (July 2016), MIT (December 2016), Toulouse (August 2017), Perimeter Institute (March 2018), Hefei (November 2018), Ohio (January 2019), Indiana (February 2019, February 2024×2), Montreal (July 2019), London (August 2019), Melbourne (October 2019, (2)), Groningen (February 2020, June 2022, May 2024), Moscow (April 2020, online), Waco (March 2022), ICERM (April 2022, May 2023), Oaxaca (October 2022), Los Angeles (November 2022, December 2024), Waterloo (October 2024), Budapest (November 2024×2), Beijing Jiaotong University (July 2024), UBC (October 2024), online (February 2025), and many other lectures in Boston University, Brandeis, Harvard, MIT, the Hebrew University, and the University of Toronto.

Outreach Lectures: (See also $\omega\varepsilon\beta$ /Talks)

1. *On Maps, Machines and Roaches*, $\omega\varepsilon\beta$ /Talks/Machines. Berkeley (October 1999×2), Tel Aviv (November 2000), Toronto (January 2005, February 2005, August 2012, August 2013, November 2014), and many other occasions.
2. *The 17 Worlds of Planar Ants*, $\omega\varepsilon\beta$ /Talks/ClassroomAdventures-1408. Jerusalem (January 2001), Toronto (February 2002, July 2014, August 2014), Canada/USA MathCamp (July 2021).
3. *The 17 Tiling Patterns: Gotta Catch 'Em All!*, $\omega\varepsilon\beta$ /Talks/Treehouse-1410. Treehouse Talks (October 2014), TCDSB (February 2015), Math Union (Toronto, October 2015), Science Rendezvous (Toronto, May 2016).
4. *Non-Commutative Gaussian Elimination and Rubik's Cube*, $\omega\varepsilon\beta$ /Talks/Cambridge-1301. Canadian Undergraduate Mathematics Conference (Toronto July 2008), Canada/USA MathCamp (Tacoma July 2009), Adams Society (Cambridge January 2013), Singapore (May 2013), and several other occasions.
5. *Dessert: Hilbert's 13th Problem, in Full Colour*, $\omega\varepsilon\beta$ /Talks/Fields-1411. Fields Institute (November 2009, November 2014), and many other occasions.
6. *The Hardest Math I've Ever Really Used*, $\omega\varepsilon\beta$ /Talks/RCI-110213. Kansas (April 2009), Royal Canadian Institute (February 2011), St. Bonaventure University (October 2011), Science Atlantic Conference (Mt. Allison University October 2012), Singapore (May 2013), Canada Math Camp (Toronto August 2013), Canadian Mathematical Society Public Lecture (Niagara Fall, December 2016), Sydney (September 2019), and several other occasions.
7. *Visualizing the Fourth Dimension, and the Simplest Thing I Don't Know About It*, $\omega\varepsilon\beta$ /Talks/ClassroomAdventures-1401. Canadian Undergraduate Mathematical Conference (Montreal, July 2013), Jerusa-

- lem (January 2014), Classroom Adventures (Toronto, January 2014).
8. *Commutators*, [ωεβ/Talks/CMU-1504](#). Toronto (January 2015), Carnegie Mellon University (April 2015).
 9. *Knots in Three and Four Dimensions*, [ωεβ/Talks/Cornell-150925/](#). Toronto (May 2015), Cornell (September 2015), Syracuse (online, [April 2021](#)), Canada/USA MathCamp (July 2021).
 10. *Nobody Solves the Quintic*, [ωεβ/Talks/Sydney-1708/](#). Sydney (August 2017), Western (online, [August 2020](#)).
 11. *My Favourite First-Year Analysis Theorem*, [ωεβ/Talks/MAASeaway-1810/](#). University of Toronto Mississauga (October 2018), Toronto (online, [June 2020](#)).
 12. *Computing the Zombian of an Unfinished Columbarium*, Knot at Lunch Seminar, Sydney March 2023.

Classes taught in the last seven years: (See also [ωεβ/classes](#))

1. *Math 1301 — Algebraic Topology* (Toronto, 2024-2025, spring term).
2. *Math 327 — Introduction to Topology* (Toronto, 2024-2025, fall term).
3. *Fast Computations in Knot Theory* (a short class at Tsuda University, June-July 2023).
4. *Math 257 — Analysis II* (Toronto, 2021-2022, all year).
5. *Math 1350 — Topics in Knot Theory* (Toronto, 2021-2022, fall term).
6. *Math 257 — Analysis II* (Toronto, 2020-2021, all year).
7. *Math 1350 — Topics in Knot Theory* (Toronto, 2020-2021, fall term).
8. *Math 327 — Introduction to Topology* (Toronto, 2018-2019, fall term).
9. *Math 1750 — Shameless Mathematica* (Toronto, 2017-2018, fall term).

Present students:

1. Ramana Murugesan, beginning Ph.D. student.
2. Kevin Santos, Ph.D. student since summer 2024, working on positive knots.
3. Daniel Martchenkov, Ph.D. student since summer 2023, working on relations between presentations of the Alexander module.
4. Jessica Liu, Ph.D. student since summer 2021, working on knot signatures. See also [ωεβ/Students/#Liu](#).

Past students: (See also [ωεβ/Students](#))

1. Leonard Afeke, Ph.D., 2024. Thesis title: *On the Gassner Invariant of Braids and String links*. See also [ωεβ/Students/#Afeke](#).
2. Daniel Martchenkov, M.Sc. project, 2023. Worked on Determinant Formulas for the Alexander Polynomial. See also [ωεβ/Students/#Martchenkov](#).
3. Jesse Frohlich, Ph.D., 2023. Thesis title: *Computing the Generating Function of a Coinvariants Map*. See also [ωεβ/Students/#Frohlich](#).
4. Sina Abbasi, M.Sc. project, 2020. Worked on w-braid groups. See also [ωεβ/Students/#Abbasi](#).
5. Travis Ens, Ph.D., 2020. Thesis title: *On Braidors: An Analogue of the Theory of Drinfel'd Associators for Braids in an Annulus*. See also [ωεβ/Students/#Ens](#).

6. David Ledvinka, undergraduate summer project in 2018-19 on an algorithm by Vertigan. See also [ωεβ/Students/#Ledvinka](#).
7. Huan Vo, Ph.D. (2018). Thesis title: *Alexander Invariants of Tangles Via Expansions*. See also [ωεβ/Students/#Vo](#).
8. Calder Morton-Ferguson, undergraduate summer project in 2016-17 on *A Visual Companion to Hatcher's Notes on Basic 3-Manifold Topology*. See also [ωεβ/Students/#MortonFerguson](#).
9. Andrey Khesin, undergraduate projects in 2015-16 on a database of ribbon knots and on a new generation of the Rolfsen table. See also [ωεβ/Students/#Khesin](#).
10. Jesse Bettencourt, M.Sc. project, 2016. Worked on torus knot fibrations. See also [ωεβ/Students/#Bettencourt](#).
11. Iva Halacheva, undergraduate projects, 2007 and 2008, M.Sc. project, 2011, and Ph.D., 2016 (jointly supervised with J. Kamnitzer). Thesis title: *Alexander Type Invariants of Tangles, Skew Howe Duality for Crystals and the Cactus Group*. See [ωεβ/Students/#Halacheva](#).
12. Oleg Chterental, M.Sc. project, 2010, and Ph.D., 2015. Thesis title: *Virtual Braids and Virtual Curve Diagram*. See also [ωεβ/Students/#Chterental](#).
13. Jonathan Zung, M.Sc. project, 2014. Worked on finite type invariants of doodles. See also [ωεβ/Students/#Zung](#).
14. Sam Selmani, M.Sc. project, 2012. Wrote paper (jointly with me) titled *Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial*. See also [ωεβ/Students/#Selmani](#).
15. Karene Chu, Ph.D., 2012. Thesis title: *Flat Virtual Pure Tangles*. See also [ωεβ/Students/#Chu](#).
16. Peter Lee, Ph.D., 2011. Thesis title: *The Pure Virtual Braid Group Is Quadratic*. See also [ωεβ/Students/#LeeP](#).
17. Zsuzsanna Dancso, Ph.D., 2011. Thesis title: *On a Universal Finite Type invariant of Knotted Trivalent Graphs*. See also [ωεβ/Students/#Dancso](#).
18. Emily Cliff, M.Sc. project, 2011. Studied the Belavin-Drinfel'd classification of Lie bialgebras. (Jointly supervised with J. Kamnitzer).
19. Qin Deng, undergraduate project on combinatorics and dynamical systems, summer 2011.
20. Louis Leung, Ph.D., 2010. Thesis title: *Classical Lie Algebra Weight Systems of Arrow Diagrams*. See also [ωεβ/Students/#Leung](#).
21. Jana Archibald, M.Sc. project, 2005 and Ph.D., 2010. Thesis title: *The Multivariable Alexander Polynomial on Tangles*. See also [ωεβ/Students/#Archibald](#).
22. Hernando Burgos, Ph.D., 2009 (formally in Bogota). Thesis title: *The Jones Polynomial and the Planar Algebra of Alternating Links*. See also [ωεβ/Students/#Burgos](#).
23. Fionntan Roukema, M.Sc. project, 2007. Studied the Goussarov-Polyak-Viro "Gauss-diagram formulas". See [ωεβ/Students/#Roukema](#).
24. Gad Naot, Ph.D., 2007. Thesis title: *The Universal sl_2 Link Homology Theory*. See also [ωεβ/Students/#Naot](#).
25. Zavosh Amir-Khosravi, undergraduate project, 2006. Participated in writing *VasCalc — A Vassiliev Invariants Calculator*.
26. Dan Carney, M.Sc. project, 2005. Computed [braid representatives and multivariable Alexander polynomials](#) of knots and links. See [ωεβ/Students/#Carney](#).
27. Siddarth Sankaran, undergraduate projects, 2005 and 2006. Wrote [several knot theory programs](#), including part of *VasCalc — A Vassiliev Invariants Calculator*. See [ωεβ/Students/#Sankaran](#).
28. Jeremy Green, undergraduate projects, 2004 and 2005. Project titles: *A Table of Virtual Knots* and *JavaKh*. See [ωεβ/Students/#GreenJ](#).

29. Emily Redelmeier, undergraduate project, 2003. Project title: *Drawing Planar Diagrams*. See [ωεβ/Students/#Redelmeier](#).
30. Stephen Green, undergraduate project, 2003. Project title: *The Planar Enumerator*. See [ωεβ/Students/#Green](#).
31. Ami Haviv, Ph.D., 2003. Thesis title: *Towards a Diagrammatic Analogue of the Reshetikhin-Turaev Link Invariants*. See [ωεβ/Students/#Haviv](#).
32. Daniel Moskovich, research project, 2002. Paper's title: *Framing and the Self-Linking Integral*. See [ωεβ/Students/#Moskovich](#).
33. Dori Eldar, M.Sc., 1999. Thesis title: *Planar Machines Web Site: An Introduction to Topology*. See [ωεβ/People/Eldar/thesis](#).
34. Avishay Vaknin, M.Sc., 1997. Thesis title: *Associahedrons and the Mac-Lane Coherence Theorem*.

Post-doctoral Supervisions:

- **Past.** Nancy Scherich, Ester Dalvit, Peter Samuelson, Dave Penneys, Karene Chu, Zsuzsanna Dancso, Daniel Moskovich, Andrew Kricker.
- **Current.** Tamara Hogan.

Funding History:

- NSF grant (with R. Bott and C. Taubes, 1992–1995).
- BSF grant, with R. Lawrence, M. Hutchings, V. Jones, and L. Rozansky (2000-2003, US\$84,000, my participation ended upon moving to Toronto in 2002).
- ISF grant, with M. Polyak (2001–2005, US\$112,000, my participation ended upon moving to Toronto in 2002).
- NSERC Discovery grant “new and newer knot invariants” (2003–2008, CA\$100,000).
- NSERC Discovery grant “knot theory and algebra” (2008–2013, CA\$140,000).
- NSERC Accelerator grant “knot theory and algebra” (2009–2012, CA\$120,000).
- NSERC Discovery grant “knot theory, higher knot theory, and algebra” (2013–2018, CA\$145,000).
- Simons Sabbatical grant (2013, CA\$129,755).
- NSERC Discovery grant “Poly-Time Knot Theory and Quantum Algebra” (2018–2024, CA\$168,000).
- Grant from the Chu Family Foundation “Quality Knot Invariants”, (2022–2024, CA\$243,100).
- Grant from the Chu Family Foundation to support a post-doctoral position for Tamara Hogan, (2024–2027, CA\$248,447).
- NSERC Discovery grant “knot theory as an excuse”, (2025–2030, CA\$170,000).

Further Activities:

- Organizer, *Expansions, Lie Algebras, and Invariants*, month-long workshop at CRM, Montreal, July 2019.
- Associate Chair for Undergraduates, University of Toronto, 2017-2019.
- Graduate Coordinator, University of Toronto, 2009-2012.
- **Founding editor** of *The Knot Atlas* (see <http://katlas.org>).
- **Founding Contributor** to KnotTheory', a computer knot theory package (see <http://katlas.org/wiki/KnotTheory>).
- 10 year **Member of the Editorial Board**, *Compositio Mathematica* (retired 2010).

[© Publications](#) [Talks](#) [Classes](#) [Students](#) [Profile](#) [KAtlas](#) [Pensieve](#) [Wiki](#) [Album](#) [Gallery](#) [Codes](#) [Misc](#) [Random](#)

Search

Dror Bar-Natan

Professor, [Department of Mathematics](#), [University of Toronto](#), [Toronto](#), [Ontario](#), [Canada](#).

Office: [Bahen Centre Room 6178](#).

E-Mail: drorbn@math.toronto.edu.

[Slow Mail](#) and [Map](#) (@ N43.659513 W79.397751).



[I'm hyper-famous! Click to see how](#)



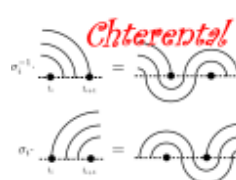
[Publications](#)



[Talks](#)



[Classes](#)



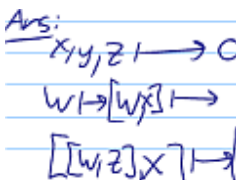
[Students](#)



[Profile](#)
(CV, statements, opinions)



[The Knot Atlas](#)



[Academic Pensieve](#)



[Wiki](#)



[Album](#)



[Image Gallery](#)



[Bible Codes](#)
(I'm out! $\frac{1}{2}$)



[Odds, Ends, Unfinished](#)

[<< A >>](#) I don't understand supersymmetry.

Online since [1994!](#)
[Copyleft Notice](#)