```
K = X[1, 4, 2, 5] X[3, 6, 4, 1] X[5, 2, 6, 3]
```

X[1, 4, 2, 5] X[3, 6, 4, 1] X[5, 2, 6, 3]

```
SetAttributes[P, Orderless];
KB0[K_] := Expand[
   Expand[
     K /. X[a_, b_, c_, d_] :→
       A P[a, d] P[b, c] + 1 / A P[a, b] P[c, d]
    ] //. {
     P[a_, b_] P[b_, c_] :→ P[a, c],
     P[a_, b_]^2 → -A^2 - 1 / A^2}
  ]
```

```
KB0[K]
```

$$-\frac{1}{A^9} + \frac{1}{A} + A^3 + A^7$$

```
KB1[pd_PD] := KB1[pd, {}, 1];
KB1[pd_PD, inside_, web_] := Module[
    {pos = First[Ordering[
        Length[Complement[List @@ #, inside]] & /@ pd]]},
    pd[[pos]] /. X[a_, b_, c_, d_] :→ KB1[
       Delete[pd, pos], Union[inside, {a, b, c, d}],
       Expand[
         web * (A P[a, d] P[b, c] + 1 / A P[a, b] P[c, d])]
        //. {P[e_, f_] P[f_, g_] :→ P[e, g],
          P[e_, _]^2 :→ P[e, e], P[e_, e_] → -A^2 - 1 / A^2}
       ]
    ];
KB1[PD[], _, web_] := Expand[web]
```