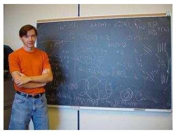
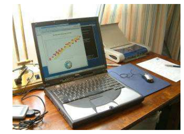
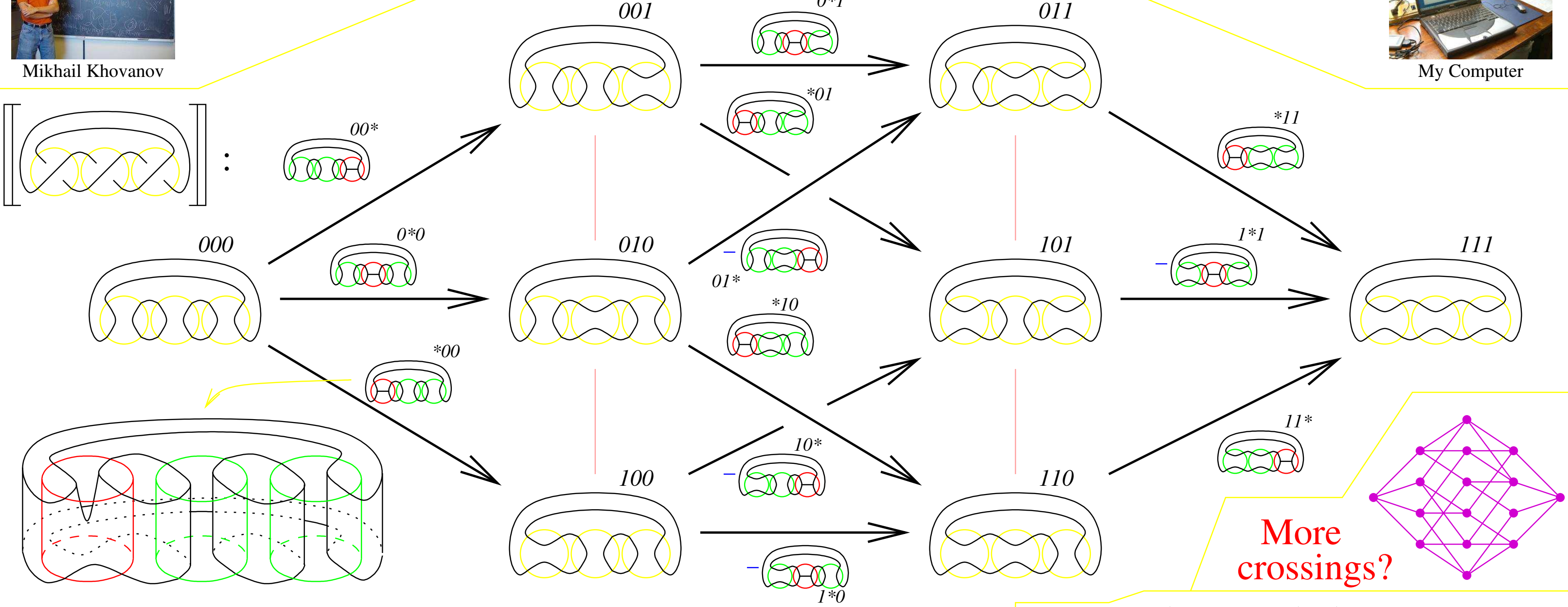
Fast Khovanov Homology Computations



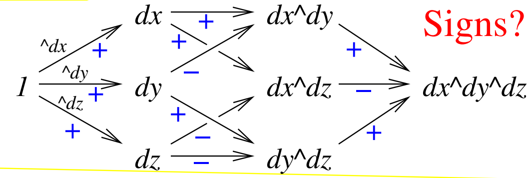
Mikhail Khovanov



My Computer

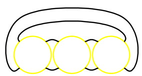
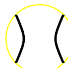
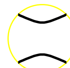


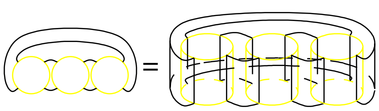
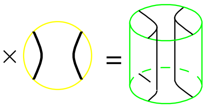
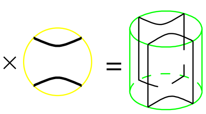
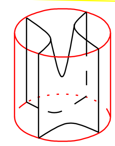
More crossings?



What is it?

A cube for each knot/link projection;

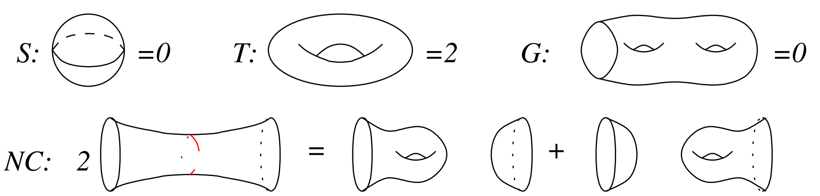
Vertices: All fillings of  with  or with .

Edges: All fillings of $I\times$  with $I\times$  or with $I\times$  and precisely one .

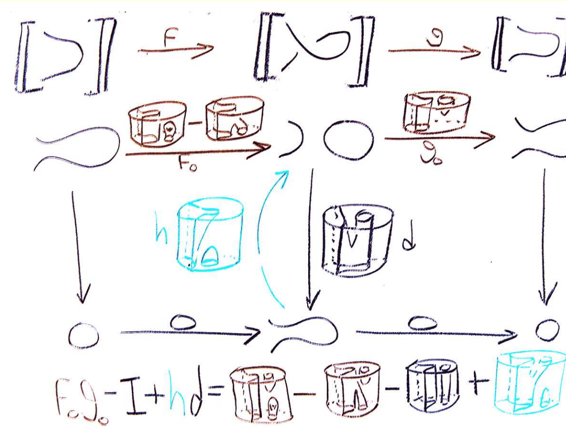
Where does it live?

In *Kom(Mat(<Cob> / {S, T, G, NC})) / homotopy*

Kom: Complexes Mat: Matrices Cob: Cobordisms <...>: Formal lin. comb.

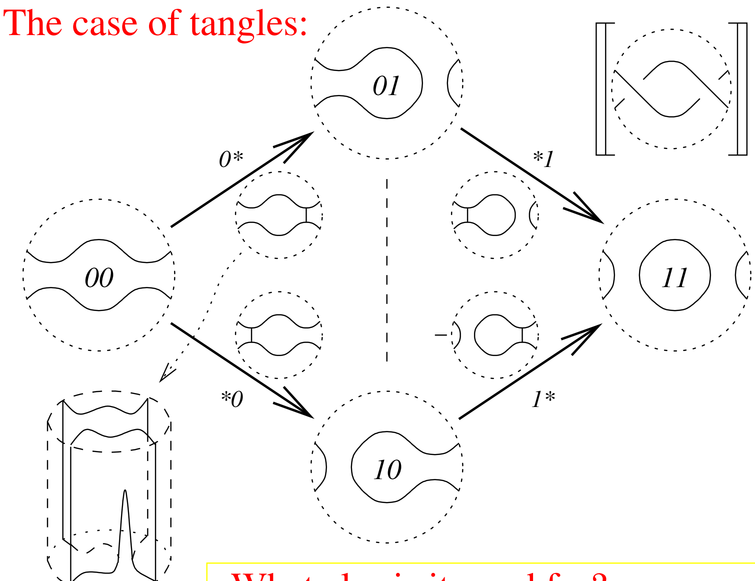


Invariance under R1

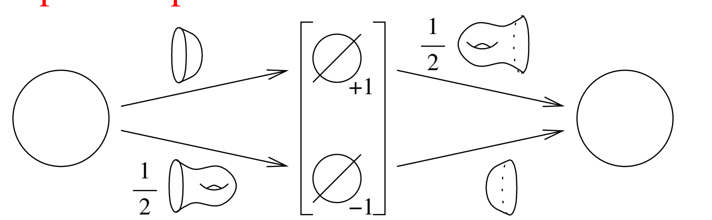


(R2, R3 are similar)

The case of tangles:



Complex simplification:



The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[\bullet]\xrightarrow{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}}\begin{bmatrix}\cdot\\ \bullet\end{bmatrix}\xrightarrow{\begin{pmatrix}\phi & \delta\\ \gamma & \epsilon\end{pmatrix}}\begin{bmatrix}\cdot\\ \bullet\end{bmatrix}\xrightarrow{\begin{pmatrix}\mu & \nu\end{pmatrix}}[\bullet]$$

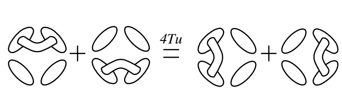
is isomorphic to the (direct sum) complex

$$[\bullet]\xrightarrow{\begin{pmatrix}0\\ \beta\end{pmatrix}}\begin{bmatrix}\cdot\\ \bullet\end{bmatrix}\xrightarrow{\begin{pmatrix}\phi & 0\\ 0 & \epsilon-\gamma\phi^{-1}\delta\end{pmatrix}}\begin{bmatrix}\cdot\\ \bullet\end{bmatrix}\xrightarrow{\begin{pmatrix}0 & \nu\end{pmatrix}}[\bullet]$$

What else is it good for?

It's local!

1. A localized relation with Kauffman's bracket.
2. Easily generalizes to surfaces, virtuals, etc.
3. Better understanding of functoriality.
4. Removing G and replacing NC with 4Tu yields a more general theory!



See also http://www.math.toronto.edu/~drorbn/Talks/GWU-050213/