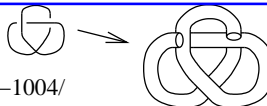


w-Knots from Z to A

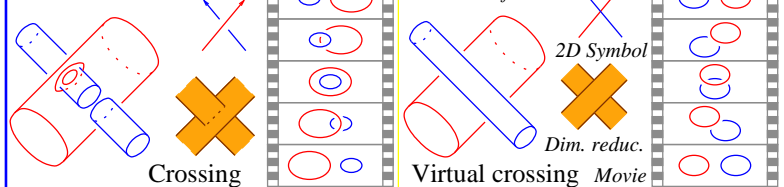
Dror Bar-Natan, Luminy, April 2010

http://www.math.toronto.edu/~drorbn/Talks/Luminy-1004/

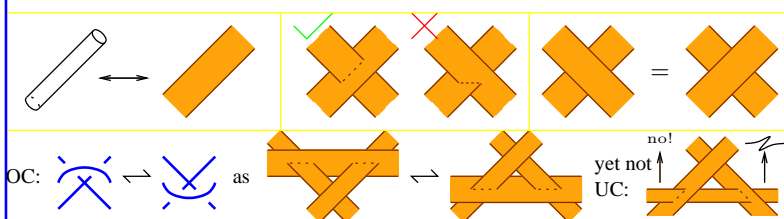


Abstract I will define w-knots, a class of knots wider than ordinary knots but weaker than virtual knots, and show that it is quite easy to construct a universal finite invariant Z of w-knots. In order to study Z we will introduce the “Euler Operator” and the “Infinitesimal Alexander Module”, at the end finding a simple determinant formula for Z . With no doubt that formula computes the Alexander polynomial A , except I don't have a proof yet.

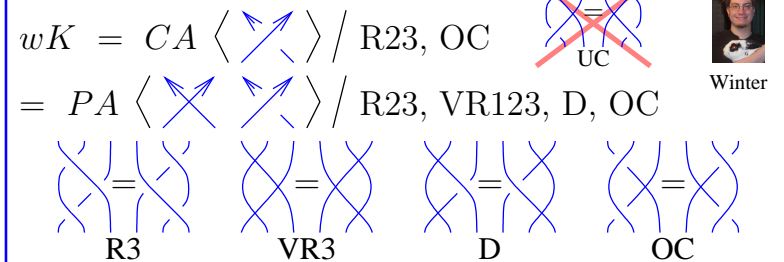
Tubes in 4D.



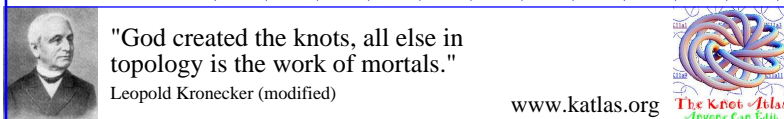
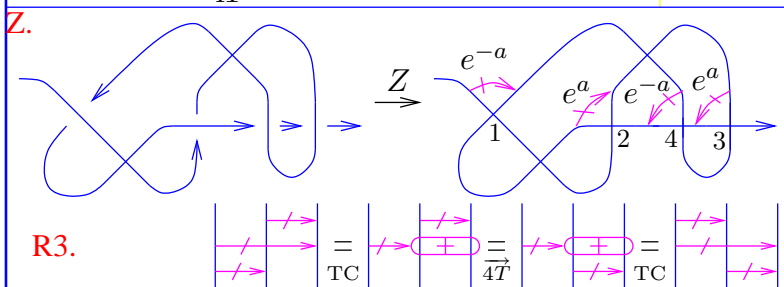
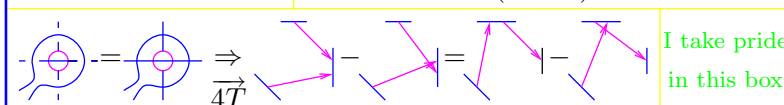
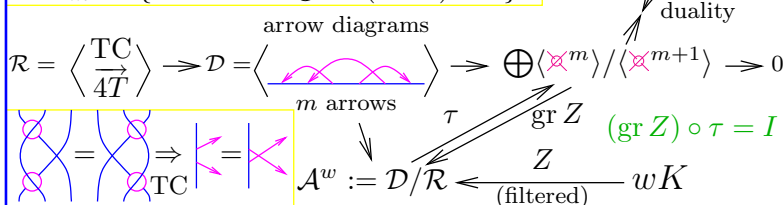
A **Ribbon 2-Knot** is a surface S embedded in \mathbb{R}^4 that bounds an immersed handlebody B , with only “ribbon singularities”; a ribbon singularity is a disk D of trasverse double points, whose preimages in B are a disk D_1 in the interior of B and a disk D_2 with $D_2 \cap \partial B = \partial D_2$, modulo isotopies of S alone.



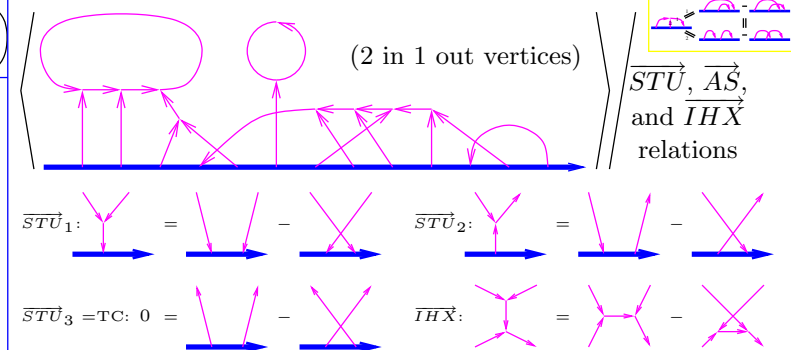
w-Knots.



The Finite Type Story. With $\bowtie := \times - \times$ set $\mathcal{V}_m := \{V : wK \rightarrow \mathbb{Q} : V(\bowtie^{>m}) = 0\}$.

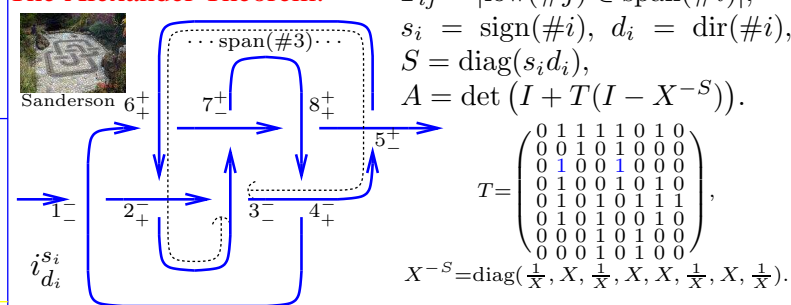


The Bracket-Rise Theorem. \mathcal{A}^w is isomorphic to



Corollaries. (1) Related to Lie algebras! (2) Only wheels and isolated arrows persist. **Habiro - can you do better?**

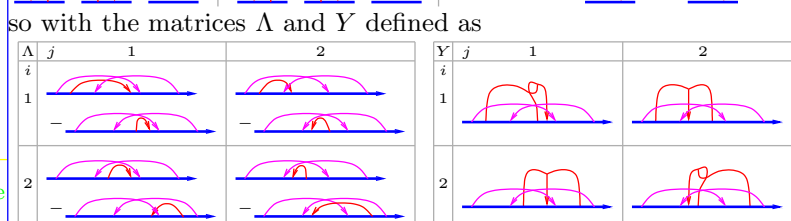
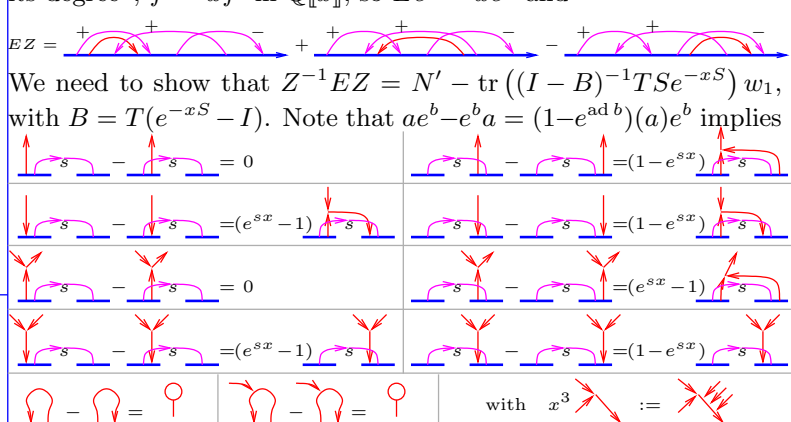
The Alexander Theorem.



Conjecture. For u-knots, A is the Alexander polynomial.

Theorem. With $w : x^k \mapsto w_k =$ (the k -wheel), $Z = N \exp_{\mathcal{A}^w}(-w(\log_{\mathbb{Q}[[x]]} A(e^x))) \pmod{w_k w_l = w_{k+l}, Z = N \cdot A^{-1}(e^x)}$

Proof Sketch. Let E be the Euler operator, “multiply anything by its degree”, $f \mapsto x f'$ in $\mathbb{Q}[[x]]$, so $E e^x = x e^x$ and



we have $EZ - N'' = \text{tr}(S\Lambda)$, $\Lambda = -BY - Te^{-xS}w_1$, and $Y = BY + Te^{-xS}w_1$. The theorem follows.

So What? • Habiro-Shima did this already, but not quite. (HS: *Finite Type Invariants of Ribbon 2-Knots, II*, Top. and its Appl. **111** (2001).)
 • New (?) formula for Alexander, new (?) “Infinitesimal Alexander Module”. Related to Lescop's arXiv:1001.4474?
 • An “ultimate Alexander invariant”: local, composes well, behaves under cabling. Ought to also generalize the multi-variable Alexander polynomial and the theory of Milnor linking numbers.
 • Tip of the Alekseev-Torossian-Kashiwara-Vergne iceberg (AT: *The Kashiwara-Vergne conjecture and Drinfeld's associators*, arXiv:0802.4300).
 • Tip of the v-knots iceberg. May lead to other polynomial-time polynomial invariants. “A polynomial's worth a thousand exponentials”.
 Also see <http://www.math.toronto.edu/~drorbn/papers/WKO/>