The Strongest Genuinely Computable Knot Invariant ~~Since~~ In 2024

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Abstract. "Genuinely computable" means we have computed it for random knots with over 300 crossings. "Strongest" means it separates prime knots with up to 15 crossings better than the less-computable HOMFLY-PT and Khovanov homology taken together. And hey, it's also meaningful and fun.
Continues Rozansky, Garoufalidis, Kricker, and Ohtsuki, joint with van der Veen.

These slides and the code within are online at ωεβ:=http://drorbn.net/ktc25

(I wish all speakers were making their slides available **before / for** their talks).

(I'll post the video there too)

A paper-in-progress is at ωεβ/Theta.

If you can, please turn your video on!

Happy birthday, dear Lou!

Lou Kauffman at MSRI, March 1991

Acknowledgement.

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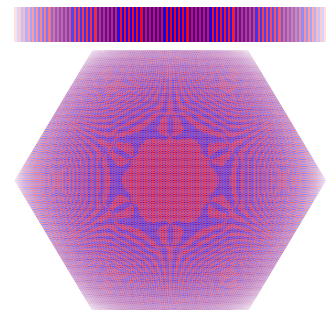
Strongest? Genuinely Computable?

Strongest.

Testing $\Theta = (\Delta, \theta)$ on prime knots up to mirrors and reversals, counting the number of distinct values (with deficits in parenthesis): (ρ_1: [Ro1, Ro2, Ro3, Ov, BV1])

	knots	(H, Kh)	(Δ, ρ_1)	$\Theta = (\Delta, \theta)$	(Δ, θ, ρ_2)	all together
reign		2005-22	2022-24	2024	2025-	
xing ≤ 10	249	248 (1)	249 (0)	249 (0)	249(0)	249 (0)
xing ≤ 11	801	771 (30)	787 (14)	798 (3)	798 (3)	798 (3)
xing ≤ 12	2,977	(214)	(95)	(19)	(10)	(10)
xing ≤ 13	12,965	(1,771)	(959)	(194)	(169)	(169)
xing ≤ 14	59,937	(10,788)	(6,253)	(1,118)	(982)	(981)
xing ≤ 15	313,230	(70,245)	(42,914)	(6,758)	(6,341)	(6,337)

Genuinely Computable. Here's Θ on a random 300 crossing knot (from [DHOEBL]). For almost every other knot invariant, that's science fiction.
Gukov: Should take 300 years if Moore's law persists.
Us: A few hours on a laptop, 0 GPUs.
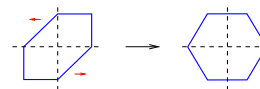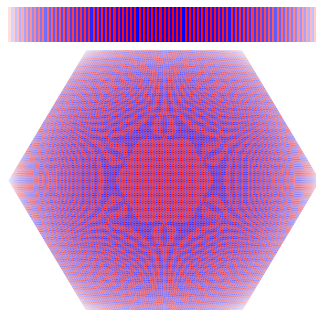
Fun. There's so much more to see in 2D pictures than in 1D ones! Yet almost nothing of the patterns you see we know how to prove. We'll have fun with that over the next few years. Would you join?

Video and more at http://www.math.toronto.edu/~drorbn/Talks/KnotTheoryCongress-2502.