

Kashaev's Signature Conjecture

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Agenda. Show and tell with signatures.

Abstract. I will display side by side two nearly identical computer programs whose inputs are knots and whose outputs seem to always be the same. I'll then admit, very reluctantly, that I don't know how to prove that these outputs are always the same. One program I wrote mostly in Bedlewo, Poland, in the summer of 2003 and as of recently I understand why it computes the Levine-Tristram signature of a knot. The other is based on the 2018 preprint *On Symmetric Matrices Associated with Oriented Link Diagrams* by Rinat Kashaev (arXiv:1801.04632), where he conjectures that a certain simple algorithm also computes that same signature.

If you can, please turn your video on! (And mic, whenever needed).

These slides and all the code within are available at <http://drorbn.net/cms21>.

(I'll post the video there too)

```

Bed[K_ , ω_] :=
Module[{t, r, KingsByArmpits, bends, faces, p, A, is},
  t = 1 - ω; r = 1 + t;
  KingsByArmpits =
  List @@ PD[K] /. x : X[{i_ , j_ , h_ , l_}] =>
  If[PositiveQ[x], X[-i, j, h, -l], X[-j, h, l, -i]];
  bends = Times @@ KingsByArmpits /.
  _[X][c_ , b_ , c_ , d_ ] => P_{c,d} P_{b,c} P_{a,c};
  faces = bends /. P_{c,d} P_{b,c} => P_{b,c};
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
  A[is, is] += If[Head[x] == X,
  
$$\begin{pmatrix} r & -t & 2t & t^* \\ -t^* & 0 & t^* & 0 \\ 2t^* & t & -t^* & t^* \\ t & 0 & -t & 0 \end{pmatrix}$$
,
  {x, KingsByArmpits}];
  MatrixSignature[A];
];

Kas[K_ , ω_] :=
Module[{u, v, KingsByArmpits, bends, faces, p, A, is},
  u = Re[ω]; v = Re[ω];
  KingsByArmpits =
  List @@ PD[K] /. x : X[{i_ , j_ , h_ , l_}] =>
  If[PositiveQ[x], X[-i, j, h, -l], X[-j, h, l, -i]];
  bends = Times @@ KingsByArmpits /.
  _[X][c_ , b_ , c_ , d_ ] => P_{c,d} P_{b,c} P_{a,c};
  faces = bends /. P_{c,d} P_{b,c} => P_{b,c};
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
  A[is, is] += If[Head[x] == X,
  
$$\begin{pmatrix} v & 1 & u & 1 \\ u & 1 & v & 1 \\ 1 & u & v & u \\ u & 1 & 1 & v \end{pmatrix}$$
,
  {x, KingsByArmpits}];
  (MatrixSignature[A] - Writhe[K]) / 2;
];

```

Why am I showing you code?

- ▶ I love code — it's fun!
- ▶ Believe it or not, it is more expressive than math-talk (though I'll do the math-talk as well, to confirm with prevailing norms).
- ▶ It is directly verifiable. Once it is up and running, you'll never ask yourself "did he misplace a sign somewhere"?

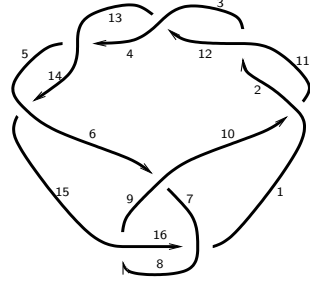
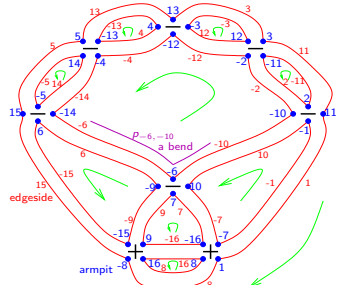
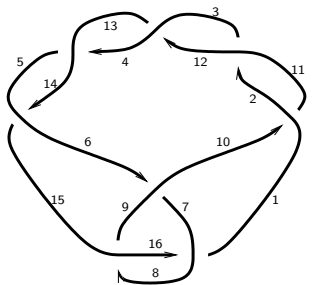
Verification.

```

Once[<< KnotTheory`
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
MatrixSignature[A_] :=
Total[Sign[Select[Eigenvalues[A], Abs[#] > 10^-12 &]]];
Writhe[K_] := Sum[If[PositiveQ[x], 1, -1], {x, List @@ PD@K}];
Sum[ω = e^{i RandomReal[{0, 2 π]}]; Bed[K, ω] == Kas[K, ω], {10},
{K, AllKnots[{3, 10}]}]
KnotTheory: Loading precomputed data in PD4Knots.
2490 True

```

Label everything!



```

PD[X[10, 1, 11, 2], X[2, 11, 3, 12], ...] {X[-1, 11, 2, -10], X[-11, 3, 12, -2], ...}

```

```

Lets run our code line by line...
PD[82] = PD[X[10, 1, 11, 2],
X[2, 11, 3, 12], X[12, 3, 13, 4],
X[4, 13, 5, 14], X[14, 5, 15, 6],
X[8, 16, 9, 15], X[16, 8, 1, 7],
X[6, 9, 7, 10]];
K = 82;

```

Video and more at <http://www.math.toronto.edu/~drorbn/Talks/CMS-2112/>