Warning. The second formula on page (-2) "**Conclusion**" is silly-wrong. A fix will be posted here soon: some of the numbers written in this handout are a bit off, yet the qualitative results remain exactly the same (namely, for finite type, 3D seems to beat 2D, with the same algorithms).

Yarn-Ball Knots

[K-OS] on October 21, 2021

Dror Bar-Natan with Itai Bar-Natan, Iva Halacheva, and Nancy Scherich

Agenda. A modest light conversation on how knots should be measured.

Abstract. Let there be scones! Our view of knot theory is biased in favour of pancakes.
Technically, if K is a 3D knot that fits in volume V (assuming fixed-width yarn), then its projection to 2D will have about $V^{4/3}$ crossings. You'd expect genuinely 3D quantities associated with K to be computable straight from a 3D presentation of K. Yet we can hardly ever circumvent this $V^{4/3} \gg V$ "projection fee". Exceptions include linking numbers (as we shall prove), the hyperbolic volume, and likely finite type invariants (as we shall discuss in detail). But knot polynomials and knot homologies seem to always pay the fee. Can we exempt them?

More at http://drorbn.net/kos21

Thanks for inviting me to speak at [K-OS]!

Most important: http://drorbn.net/kos21

See also arXiv:2108.10923.

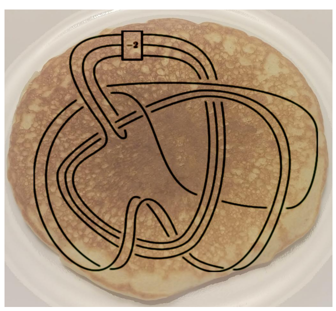
If you can, please turn your video on! (And mic, whenever needed).

A recurring question in knot theory is "do we have a 3D understanding of our invariant?"

- See Witten and the Jones polynomial.
- See Khovanov homology.

I'll talk about my perspective on the matter...

We often think of knots as planar diagrams. 3-dimensionally, they are embedded in "pancakes".



Knot by Lisa Piccirillo, pancake by DBN

But real life knots are 3D!

A Yarn Ball

'Connector' by Alexandra Griess and Jorel Heid (Hamburg, Germany). Image from www.waterfrontbia.com/ice-breakers-2019-presented-by-ports/.

The difference matters when

- We make statements about "random knots".
- We figure out computational complexity.

Let's try to make it quantitative...

Video and more at http://www.math.toronto.edu/~drorbn/Talks/KOS-211021/