

Strand Operations. c for contract, mc for magnetic contract:

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 $c_{i,j}@t : \Sigma_B[\{l_{i,j}, i, r_{i,j}\}, \{j, \_, \_ \}] [ \_ ] :=$   

 $t // GT_{j, First@\{r_i, l_i\}} // Cordon_j$ 
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 $c_{i,j}@t : \Sigma_B[\{ \_, i, j, \_ \}, \_ ] [ \_ ] := Cordon_j@t$   

 $c_{i,j}@t : \Sigma_B[\{j, \_, \_, i\}, \_ ] [ \_ ] := Cordon_j@t$   

 $c_{i,j}@t : \Sigma_B[\{ \_, j, i, \_ \}, \_ ] [ \_ ] := Cordon_i@t$   

 $c_{i,j}@t : \Sigma_B[\{i, \_, \_, j\}, \_ ] [ \_ ] := Cordon_i@t$ 
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 $mc[\mathcal{E}] := \mathcal{E} //$   

 $t : \Sigma_B[\{ \_, i, \_ \}, \{ \_, j, \_ \}, \_ ] [ \_ ] |$   

 $\Sigma_B[\{ \_, i, j, \_ \}, \_ ] [ \_ ] | \Sigma_B[\{j, \_, \_, i\}, \_ ] [ \_ ] ;$   

 $i + j = 0 \Rightarrow c_{i,j}@t$ 
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The Crossings (and empty strands).

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 $Kas@P_{i,j} := CF@ \Sigma_B[\{i,j\}] [0, PQ[\{ \}, 0]] ;$ 
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 $TL@P_{i,j} := CF@ \Sigma_B[\{i,j\}] [0, PQ[\{ \}, 0]]$ 
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```
 $Kas[x : X[i_, j_, k_, l_]] :=$   

 $Kas@If[PositiveQ[x], X_{-i,j,k,-l}, \bar{X}_{-j,k,l,-i}] ;$   

 $Kas[x : X | \bar{X}]_{fs} := Module[\{v = 2u^2 - 1, p, \gamma s, m\},$   

 $\gamma s = \gamma_{\#} \& /@ \{fs\}; p = (x == X);$   

 $m = If[p, \begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}, -\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}];$   

 $CF@ \Sigma_B[\{fs\}] [If[p, -1, 1], PQ[\{ \}, \gamma s^* . m . \gamma s]]]$ 
```

```
 $TL[x : X[i_, j_, k_, l_]] :=$   

 $TL@If[PositiveQ[x], X_{-i,j,k,-l}, \bar{X}_{-j,k,l,-i}] ;$   

 $TL[x : X | \bar{X}]_{fs} := Module[\{t = 1 - \omega, r, \gamma s, m\},$   

 $r = t + t^*; \gamma s = \gamma_{\#} \& /@ \{fs\};$   

 $m = If[x == X,$   

 $\begin{pmatrix} -r & -t & 2t & t^* \\ -t^* & 0 & t^* & 0 \\ 2t^* & t & -r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}, \begin{pmatrix} r & -t & -2t^* & t^* \\ -t^* & 0 & t^* & 0 \\ -2t & t & r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}];$   

 $CF@ \Sigma_B[\{fs\}] [0, PQ[\{ \}, \gamma s^* . m . \gamma s]]]$ 
```

Evaluation on Tangles and Knots.

```
 $Kas[K_] := Fold[mc[\#1 \oplus \#2] \&, \Sigma_B[] [0, PQ[\{ \}, 0]],$   

 $List@@(Kas /@ PD@K)] ;$   

 $KasSig[K_] := Expand[Kas[K] [1]] / 2]$ 
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```
 $TL[K_] :=$   

 $Fold[mc[\#1 \oplus \#2] \&, \Sigma_B[] [0, PQ[\{ \}, 0]],$   

 $List@@(TL /@ PD@K)] / .$   

 $\theta[c_+ + u] /; Abs[c] \geq 1 \Rightarrow \theta[c];$   

 $TLSig[K_] := TL[K] [1]$ 
```

Reidemeister 3.

$R3L = PD[X_{-2,5,4,-1}, X_{-3,7,6,-5},$

$X_{-6,9,8,-4}] ;$

$R3R = PD[X_{-3,5,4,-2}, X_{-4,6,8,-1},$

$X_{-5,7,9,-6}] ;$

$\{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R\}$

$\{True, True\}$

Kas@R3L

$$2\theta(u - \frac{1}{2}) - 2\theta(u + \frac{1}{2}) - 2$$

$\bar{\gamma}_{-3}$	$\bar{\gamma}_7$	$\bar{\gamma}_9$	$\bar{\gamma}_8$	$\bar{\gamma}_{-1}$	$\bar{\gamma}_{-2}$
$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$
$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$

Reidemeister 2.

$TL@PD[X_{-2,4,3,-1}, \bar{X}_{-4,6,5,-3}]$

$$\theta$$

	1	0	-1	0
$(\bar{\gamma}_{-2} \quad \bar{\gamma}_6 \quad \bar{\gamma}_5 \quad \bar{\gamma}_{-1})$				
$\bar{\gamma}_{-2}$	0	0	0	0
$\bar{\gamma}_6$	0	0	0	0
$\bar{\gamma}_5$	0	0	0	0
$\bar{\gamma}_{-1}$	0	0	0	0

$\{TL@PD[X_{-2,4,3,-1}, \bar{X}_{-4,6,5,-3}] == GT_{5,-2}@TL@PD[P_{-1,5}, P_{-2,6}],$
 $Kas@PD[X_{-2,4,3,-1}, \bar{X}_{-4,6,5,-3}] == GT_{5,-2}@Kas@PD[P_{-1,5}, P_{-2,6}]\}$
 $\{True, True\}$

Reidemeister 1.

$\{TL@PD[X_{-3,3,2,-1}] == TL@P_{-1,2},$

$Kas@PD[X_{-3,3,2,-1}] == Kas@P_{-1,2}\}$

$\{True, True\}$

A Knot.

$f = TLSig[Knot[8, 5]]$

$2\theta[-\frac{\sqrt{3}}{2} + u] - 2\theta[\frac{\sqrt{3}}{2} + u] -$

$2\theta[u - \text{ArcCos}[-0.630957344480193]] + 2\theta[u - \text{ArcCos}[0.630957344480193]]$

$Plot[f, \{u, -1, 1\}]$

