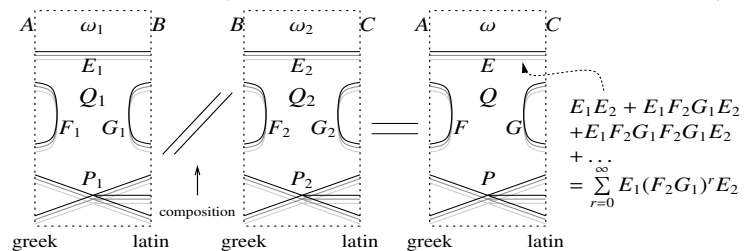


**So What?** If  $V$  is a representation, then  $V^{\otimes n}$  explodes as a function of  $n$ , while in **DoPeGDO** up to a fixed power of  $\epsilon$ , the ranks of  $\text{mor}(A \rightarrow B)$  grow polynomially as a function of  $|A|$  and  $|B|$ .

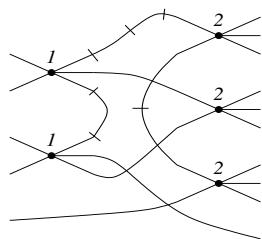
**Compositions.** In  $\text{mor}(A \rightarrow B)$ ,

$$Q = \sum_{i \in A, j \in B} E_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in A} F_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in B} G_{ij} \zeta_i \zeta_j,$$

and so (remember,  $e^x = 1 + x + xx/2 + xxx/6 + \dots$ )



where  $\bullet E = E_1(I - F_2 G_1)^{-1} E_2$ .  
 $\bullet F = F_1 + E_1 F_2 (I - G_1 F_2)^{-1} E_1^T$ .  
 $\bullet G = G_2 + E_2^T G_1 (I - F_2 G_1)^{-1} E_2$ .  
 $\bullet \omega = \omega_1 \omega_2 \det(I - F_2 G_1)^{-1}$ .  
 $\bullet P$  is computed as the solution of a messy PDE or using “connected Feynman diagrams” (yet we’re still in pure algebra!). Docility is preserved.



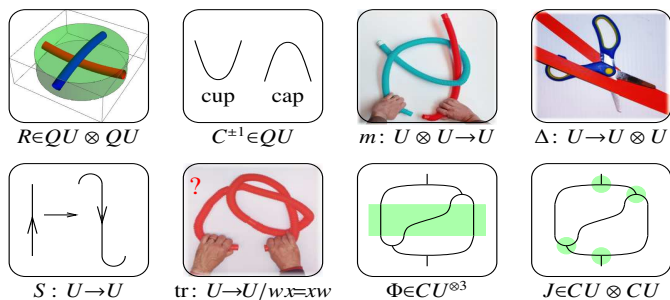
**Questions.** • Are there QFT precedents for “two-step Gaussian integration”?

• In QFT, one saves even more by considering “one-particle-irreducible” diagrams and “effective actions”. Does this mean anything here?

• Understanding  $\text{Hom}(\mathbb{Q}[z_A] \rightarrow \mathbb{Q}[z_B])$  seems like a good cause. Can you find other applications for the technology here?

$$\left( \begin{aligned} QU &= \mathcal{U}_h(sl_{2+}^\epsilon) = A(y, b, a, x) [\hbar] \text{ with } [a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, \\ [a, y] &= -y, [b, x] = \epsilon x, \text{ and } xy - qyx = (1 - AB)/\hbar, \text{ where } q = e^{\hbar \epsilon}, A = e^{-\hbar \epsilon a}, \\ \text{and } B &= e^{-\hbar b}. \text{ Also } \Delta(y, b, a, x) = (y_1 + B_1 y_2, b_1 + b_2, a_1 + a_2, x_1 + A_1 x_2), \\ S(y, b, a, x) &= (-B^{-1} y, -b, -a, -A^{-1} x), \text{ and } R = \sum \hbar^{j+k} y^j b^k \otimes a^j x^k / j! k! q^j. \end{aligned} \right)$$

**Theorem.** Everything of value regrading  $U = CU$  and/or its quantization  $U = QU$  is **DoPeGDO**:



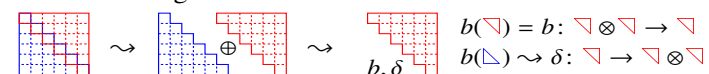
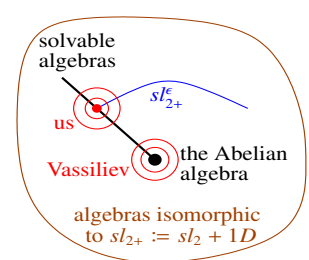
also Cartan’s  $\theta$ , the Dequantizer, and more, and all of their compositions.

**DoPeGDO Footnotes.** Each variable has a “weight”  $\in \{0, 1, 2\}$ , and always  $\text{wt } z_i + \text{wt } \zeta_i = 2$ .

- †1. Really, “weight-graded finite sets”  $A = A_0 \sqcup A_1 \sqcup A_2$ .
- †2. Really, a power series in the weight-0 variables<sup>†5</sup>.
- †3. The weight of  $Q$  must be 2, so it decomposes as  $Q = Q_{20} + Q_{11}$ . The coefficients of  $Q_{20}$  are rational numbers while the coefficients of  $Q_{11}$  may be weight-0 power series<sup>†5</sup>.
- †4. Setting  $\text{wt } \epsilon = -2$ , the weight of  $P$  is  $\leq 2$  (so the powers of the weight-0 variables are not constrained)<sup>†5</sup>.
- †5. In the knot-theoretic case, all weight-0 power series are rational functions of bounded degree in the exponentials of the weight-0 variables.
- †6. There’s also an obvious product  $\text{mor}(A_1 \rightarrow B_1) \times \text{mor}(A_2 \rightarrow B_2) \rightarrow \text{mor}(A_1 \sqcup A_2 \rightarrow B_1 \sqcup B_2)$ .

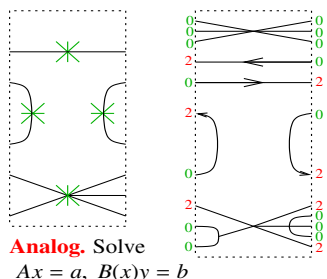
**Solvable Approximation.** In **4D Metrized Lie Algebras**

$sl_n$ , half is enough! Indeed  $sl_n \oplus a_{n-1} = \mathcal{D}(\nabla, b, \delta)$ . Now define  $sl_{n+}^\epsilon := \mathcal{D}(\nabla, b, \epsilon \delta)$ . Schematically, this is  $[\nabla, \nabla] = \nabla$ ,  $[\Delta, \Delta] = \epsilon \Delta$ , and  $[\nabla, \Delta] = \Delta + \epsilon \nabla$ . The same process works for all semi-simple Lie algebras, and at  $\epsilon^{k+1} = 0$  always yields a solvable Lie algebra.



**Full DoPeGDO.** Compute compositions in two phases:

- A 1-1 phase over the ring of power series in the weight-0 variables, in which the weight-2 variables are spectators.
- A (slightly modified) 2-0 phase over  $\mathbb{Q}$ , in which the weight-1 variables are spectators.



knot diag	$n_k^+$ $(\rho_1^+)^+$	Alexander’s $\omega^+$	genus / ribbon unknotting # / amphi?	knot diag	$n_k^+$ $(\rho_1^+)^+$	Alexander’s $\omega^+$	genus / ribbon unknotting # / amphi?	knot diag	$n_k^+$ $(\rho_1^+)^+$	Alexander’s $\omega^+$	genus / ribbon unknotting # / amphi?
	$0_1^+$ 0	1	0 / ✓ 0 / ✓		$3_1^+$ $T$	$T-1$	1 / ✗ 1 / ✗		$4_1^+$ 0	$3-T$	1 / ✗ 1 / ✓
	$5_1^+$ $2T^3 + 3T$	$T^2 - T + 1$	2 / ✗ 2 / ✗		$5_2^+$ $5T - 4$	$2T - 3$	1 / ✗ 1 / ✗		$6_1^+$ $T - 4$	$5 - 2T$	1 / ✓ 1 / ✗
	$6_2^+$ $T^3 - 4T^2 + 4T - 4$	$-T^2 + 3T - 3$	2 / ✗ 1 / ✗		$6_3^+$ 0	$T^2 - 3T + 5$	2 / ✗ 1 / ✓		$7_1^+$ $3T^5 + 5T^3 + 6T$	$T^3 - T^2 + T - 1$	3 / ✗ 3 / ✗

**Conclusion.** There are lots of poly-time-computable well-behaved near-Alexander knot invariants: • They extend to tangles with appropriate multiplicative behaviour. • They have cabling and strand reversal formulas.

$\omega\epsilon\beta/\text{akt}$   
The invariant for  $sl_{2+}^\epsilon / (\epsilon^2 = 0)$  (prior art:  $\omega\epsilon\beta/\text{Ov}$ ) attains 2,883 distinct values on the 2,978 prime knots with  $\leq 12$  crossings. HOMFLY-PT and Khovanov homology together attain only 2,786 distinct values.