Loading KnotTheory`

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<< KnotTheory`
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Loading KnotTheory` version
  of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.
```

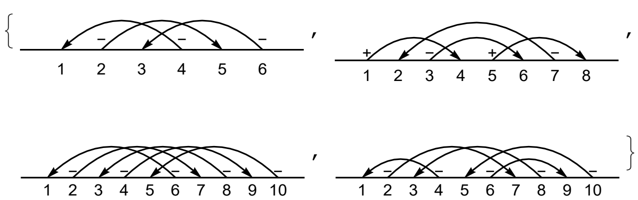
Gauss Diagram Utilities

```
GD[g_GD] := g;
GD[L_] := GD @@ PD[L] /.
   X[i_, j_, k_, l_] :> If[PositiveQ@X[i, j, k, l],
     Ap_{l,i}, Am_{j,i}];
Draw[g_GD] := Module[{n = Max@Cases[g, _Integer, ∞]},
  Graphics[{
    Line[{{0, 0}, {n + 1, 0}}],
    List @@ g /. (ah_)_{i_,j_} :> {
       Arrow[BezierCurve[{{i, 0}, {i + j, Abs[j - i]} / 2,
          {j, 0}}]],
       Text[ah /. {Ap → "+", Am → "-"}, {i, 0.3}]},
    Table[Text[i, {i, -0.5}], {i, n}]}]]
```

Some Gauss Diagrams

```
Draw /@ GD /@ AllKnots@{3, 5}
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.



Some Gauss Diagrams, 2

```
GD /@ AllKnots@{3, 5}
```

$\{GD[Am_{4,1}, Am_{6,3}, Am_{2,5}], GD[Ap_{1,4}, Ap_{5,8}, Am_{3,6}, Am_{7,2}], GD[Am_{6,1}, Am_{8,3}, Am_{10,5}, Am_{2,7}, Am_{4,9}], GD[Am_{4,1}, Am_{8,3}, Am_{10,5}, Am_{6,9}, Am_{2,7}]\}$

V_2 Definition

```
CF[g_GD] := Sort[
   g /. Thread[Sort@Cases[g, _Integer, ∞] →
      Range[2 Length[g]]]];
PV[F_GD, g_GD] /; Length[F] > Length[g] := 0;
PV[F_GD, g_GD] /; Length[F] < Length[g] := Sum[
   PV[F, γ], {γ, Subsets[g, {Length[F]}]}];
PV[F_GD, g_GD] /; Length[F] == Length[g] := If[
   CF[F] === CF[g /. Ap | Am → A], (-1)^Count[g, Am__], 0];
V2[g_] := V2[g] = PV[GD[A_{3,1}, A_{2,4}], GD[g]];
```

Computing V_2

```
Format[Knot[n_, k_]] := n_k;
Table[K → V2[K], {K, AllKnots@{3, 7}}]
```

$\{3_1 \to 1, 4_1 \to -1, 5_1 \to 3, 5_2 \to 2, 6_1 \to -2, 6_2 \to -1, 6_3 \to 1, 7_1 \to 6, 7_2 \to 3, 7_3 \to 5, 7_4 \to 4, 7_5 \to 4, 7_6 \to 1, 7_7 \to -1\}$

V_3 Definition

```
PV[F1_ + F2_, g_] := PV[F1, g] + PV[F2, g];
PV[c_ * F_GD, g_] := c PV[F, g];
ρ_k_[g_] := g /. i_Integer :> Mod[i - k, 2 Length@g, 1];
F3 = Sum_{k=0}^{5} (3 ρ_k@GD[A_{1,5}, A_{4,2}, A_{6,3}] + 2 ρ_k@GD[A_{1,4}, A_{5,2}, A_{3,6}]);
V3[K_] := V3[K] = PV[F3, GD@K] / 6;
```

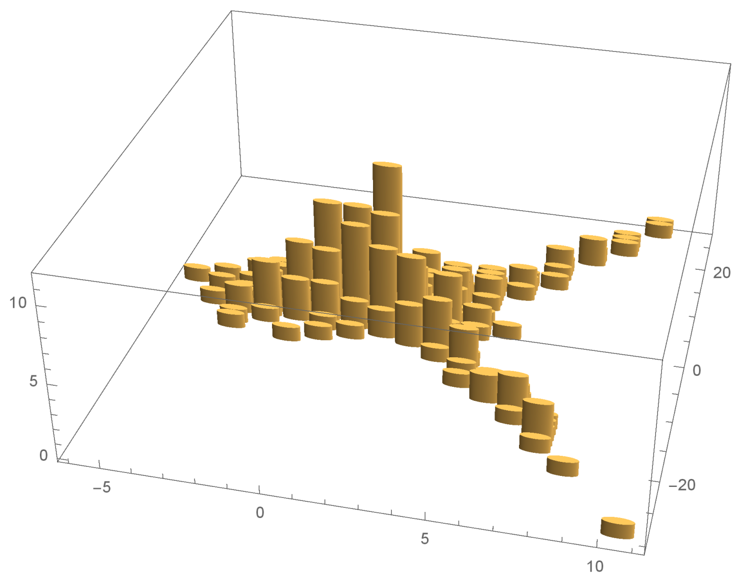
Computing V_3

```
Table[K → V3[K], {K, AllKnots@{3, 7}}]
```

$\{3_1 \to -1, 4_1 \to 0, 5_1 \to -5, 5_2 \to -3, 6_1 \to 1, 6_2 \to 1, 6_3 \to 0, 7_1 \to -14, 7_2 \to -6, 7_3 \to 11, 7_4 \to 8, 7_5 \to -8, 7_6 \to -2, 7_7 \to -1\}$

Willerton's Fish

```
Histogram3D[
 Table[{V2[K], V3[K]}, {K, AllKnots@{3, 10}}],
 {1}]
```



Gassner Utilities

```
G[λ_]_{a_,b_} := ∂_{t_a,h_b} λ;
G /: Factor[G[λ_]] :=
  G[Collect[λ, h_, Collect[#, t_, Factor] &]];
Format@γ_G := Module[{S = Union@Cases[γ, (h | t)_a_ :> a, ∞]},
   Table[γ_{a,b}, {a, S}, {b, S}] // MatrixForm] ;
```

The Gassner Program

```
G /: G[λ1_] G[λ2_] := G[λ1 + λ2];
m_{a_,b_→c_}[G[λ_]] := Module[{α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ, μ},
   ( α β θ       ( ∂_{t_a,h_a}λ  ∂_{t_a,h_b}λ  ∂_{t_a}λ
     γ δ ϵ    =    ∂_{t_b,h_a}λ  ∂_{t_b,h_b}λ  ∂_{t_b}λ
     ϕ ψ Ξ )       ∂_{h_a}λ      ∂_{h_b}λ      λ       ) /. (t | h)_{a|b} → 0;
   μ = 1 - β;
   G[Tr[(t_c; 1)^T . (γ + α δ / μ   ϵ + δ θ / μ ; ϕ + α ψ / μ   Ξ + ψ θ / μ) . (h_c; 1)]] /. T_{a|b} → T_c //
    Factor];
Rp_{a_,b_} := G[Tr[(t_a; t_b)^T . (1  1 - T_a ; 0  T_a) . (h_a; h_b)]];
Rm_{a_,b_} := Rp_{a,b} /. T_a → 1 / T_a;
```

The Gauss-Gassner-Program

```
GG[g_GD, k_, F_, BB_] :=
  Module[{n = 2 Length@g + Length@BB, y, cuts, rr, γ0, γ},
   γ0 = G[t_{n+1} h_{n+1}] Times @@ g /. {Ap → Rp, Am → Rm};
   γ0 *= G[Sum[β_{a,b} t_a h_b, {a, BB}, {b, BB}]];
   Sum[ γ = γ0;
    cuts = Cases[y, _Integer, ∞] ⋃ {n + 1};
    rr = Thread[cuts → Range[Length@cuts]];
    Do[If[! MemberQ[cuts, j], γ = γ // m_{j,j+1→j+1}], {j, n}];
    F[y /. rr, γ /. (v_)_a_ :> v_{a/.rr}],
    (*over*) {y, Subsets[List @@ g, k]}]];
GG[g_GD, k_, F_] := GG[g, k, F, {}];
```

Video and more at http://www.math.toronto.edu/~drorbn/Talks/NCSU-1604/