

## The Hardest Math I've Ever Really Used, 2



### Fermat's Principle

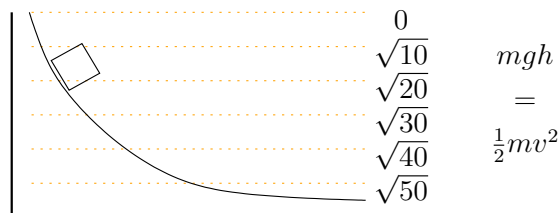
$$c \sim 300,000$$

$$c \sim 250,000$$



Picture credits: Mona: Leonardo; Al Gore: Futurama; Map 1: en.wikipedia.org/wiki/Greenhouse-gas; Smokestacks: ghuapcd.org/complaint.htm; Penguin: brentpabst.com/bp/2007/12/15/BrentGoesPenguin.aspx; Map 2: flightpedia.org; Segway: co2calculator.wordpress.com/2008/10; Lobachevsky: en.wikipedia.org/wiki/Nikolai.Lobachevsky; Eschers: www.josleys.com/show\_gallery.php?galid=325;

### The Brachistochrone



Bernoulli on Newton. "I recognize the lion by his paw".

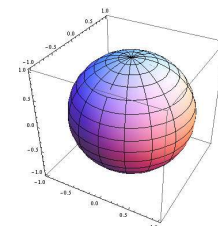
### Flatlanders airline route map



**The Least Action Principle.** Everywhere in physics, a system goes from  $A$  to  $B$  along the path of least action.

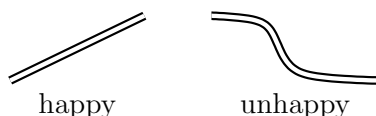
With small print for quantum mechanics.

```
ParametricPlot3D[{
  Sin[u] Cos[v],
  Sin[u] Sin[v],
  Cos[u]
}, {u, 0, π}, {v, 0, 2 π}]
```

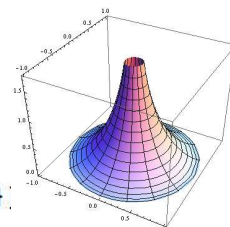


### The Happy Segway Principle

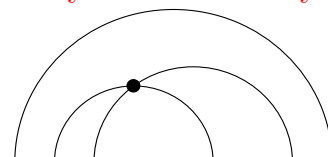
A Segway is happy iff both its wheels are



```
ParametricPlot3D[{
  Sech[u] Cos[v],
  Sech[u] Sin[v],
  u - Tanh[u]
}, {u, 0, e}, {v, 0, 2 π}]
```



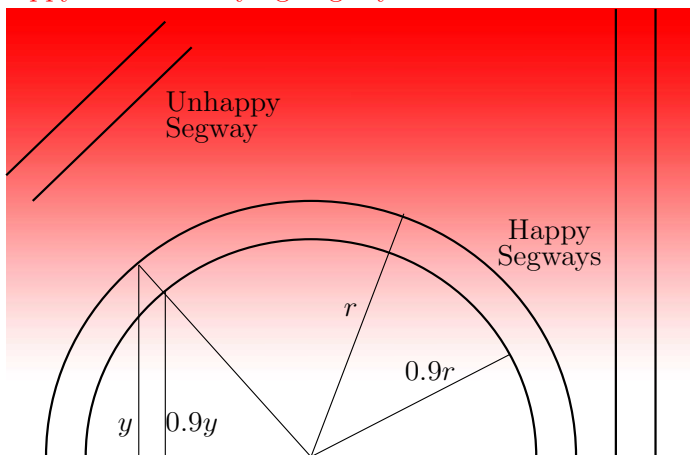
### The Bolyai-Lobachevsky Plane



Two parallels through one point

**Further Fun Facts.** • In small scale,  $\pi^H \rightarrow \pi^E$ . In large scale,  $\pi^H \rightarrow \infty$ . • The sum of the angles of a triangle is always less than  $\pi$ . In fact,  $\text{sum} + \text{area} = \pi$ , so the largest possible area of a triangle is  $\pi$ . • If your friend walks away, she'll drop out of sight before you know it. • There are so many places just a stone throw away! But you'd better remember your way back well!

### Happy camera-carrying Segways above the Mona Plane



The Mona Plane

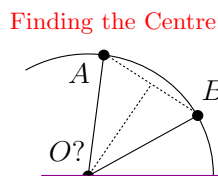


### The Actual Code

```
p3.y = p2.y + b*x3p;
x = p1.x-p2.x; y = p1.y-p2.y;
d1 = p1.d; d2 = p2.d;
norm = sqrt(x*x + y*y);
a = x/norm; b = y/norm;
x1p = a*x + b*y;
x0 = (x1p + (d1*d1-d2*d2)/x1p)/2;
r = sqrt((x1p-x0)*(x1p-x0)+d1*d1);
x1pp = (x1p-x0)/r; x2pp = -x0/r;
theta1 = acos(x1pp);
theta2 = acos(x2pp);
t1 = log(tan(theta1/2));
t2 = log(tan(theta2/2));
t3 = t1 + s*(t2-t1);
theta3 = 2*atan(exp(t3));
x3pp = cos(theta3);
d3pp = sin(theta3);
x3p = x0 + r*x3pp;
p3.d = r*d3pp;
p3.x = p2.x + a*x3p;
```

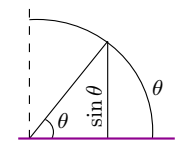
Ops used. +, -, ×, ÷, √, cos, sin, tan, arccos, arctan, log, exp.

Some further basic geometry also occurs:



### Finding the Centre

### Parametrization



$$\theta'(t) = \sin \theta(t)$$

$$\Downarrow$$

$$\theta = 2 \arctan e^t$$