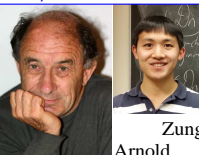


# Finite Type Invariants of Doodles, 1



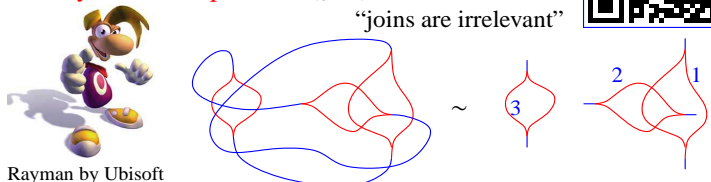
**Abstract.** I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals".

**An unfinished project!**

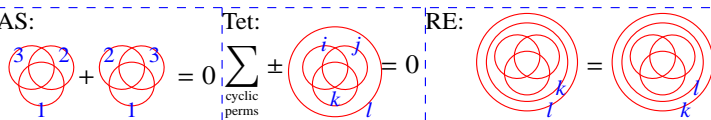
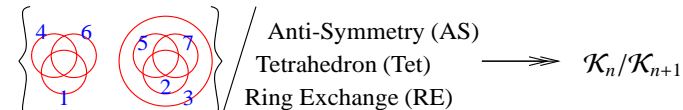
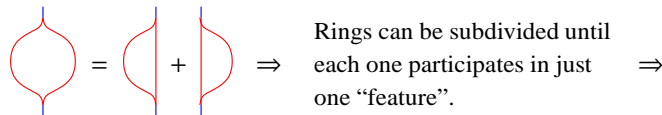


## Chord Diagrams and an Upper Bound on $\mathcal{K}_n/\mathcal{K}_{n+1}$

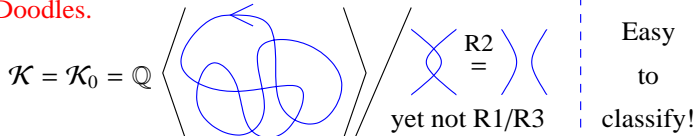
**The Rayman Principle.** In  $\mathcal{K}_n/\mathcal{K}_{n+1}$ , "joins are irrelevant"



**The Subdivision Relations.** In  $\mathcal{K}_n/\mathcal{K}_{n+1}$ ,



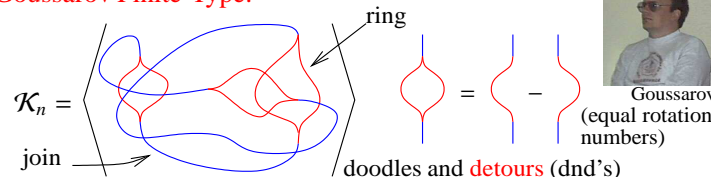
## Doodles.



**Prior Art.** Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness"  $St$  invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.



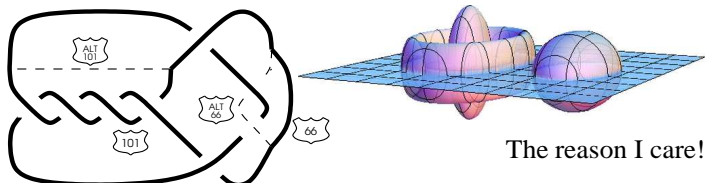
## Goussarov Finite-Type.



**Def.**  $V$  is of type  $n$  if it vanishes on  $\mathcal{K}_{n+1}$ .  $(\mathcal{K}_0/\mathcal{K}_{n+1})^* \leftrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

## Knots in 3D.

## 2-Knots in 4D.



**Goals.** • Describe  $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$  using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if  $\mathcal{A}_n$  has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's  $St$  and the Merkov invariants integrate in.

## The Primary Snippet.

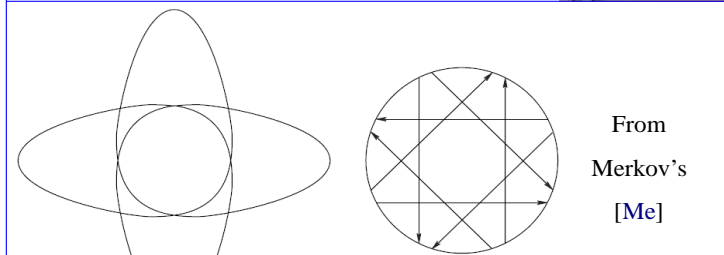
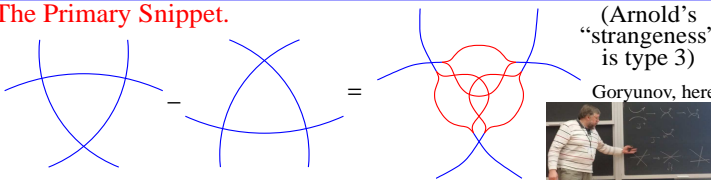
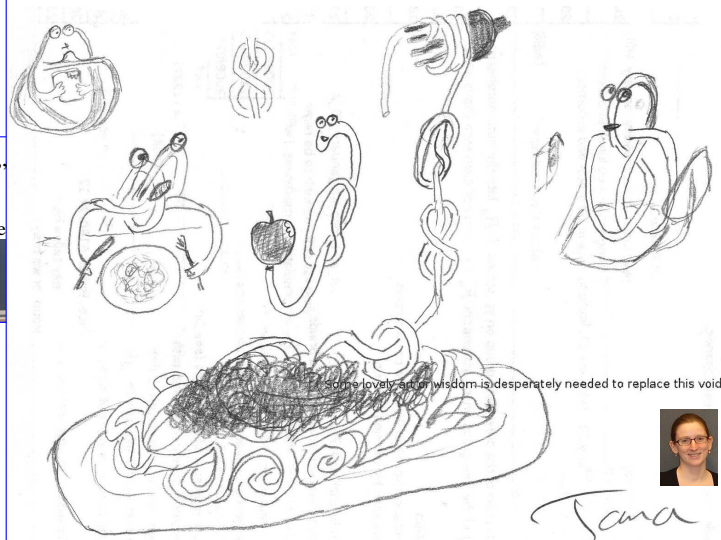
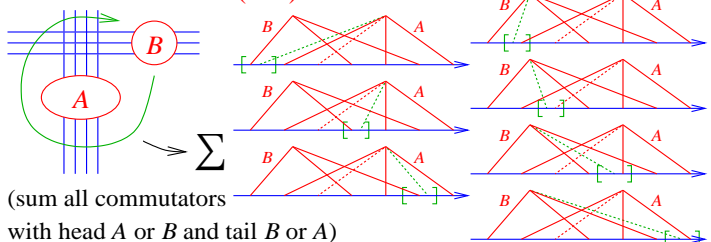


Figure 3. A non-trivial 1-doodle and its arrow diagram

## "Chord Diagrams".



## "Multi-Commutator" (MC) Relations.



Doodles by my former student Jana Archibald



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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