

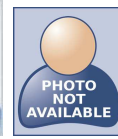
A Partial Reduction of BF Theory to Combinatorics, 1

Abstract. I will describe a **semi-rigorous** reduction of perturbative BF theory (Cattaneo-Rossi [CR]) to computable combinatorics, in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. **Weak** this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting. (good news in **highlight**)

The BF Feynman Rules. For an edge e , let Φ_e be its direction, in S^3 or S^1 . Let ω_3 and ω_1 be volume forms on S^3 and S^1 . Then for a 2-link $(K_i)_{i \in T}$,



Cattaneo



Rossi



$$\zeta = \log \sum_{\text{diagrams } D} \frac{[D]}{|\text{Aut}(D)|} \int_{\mathbb{R}^2} \dots \int_{\mathbb{R}^2} \int_{\mathbb{R}^4} \dots \int_{\mathbb{R}^4} \prod_{e \in D} \Phi_e^* \omega_3 \prod_{e \in D} \Phi_e^* \omega_1$$

S -vertices M -vertices

is an invariant in $CW(FL(T)) \rightarrow CW(T)/\sim$, "symmetrized cyclic words in T ".

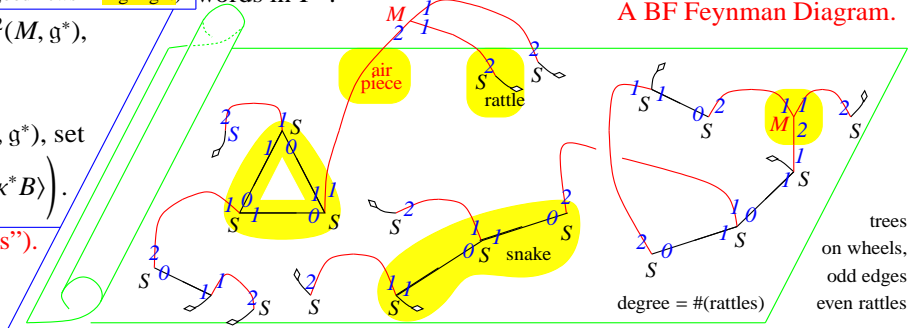
BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, \mathfrak{g})$, $B \in \Omega^2(M, \mathfrak{g}^*)$,

$$S(A, B) := \int_M \langle B, F_A \rangle.$$

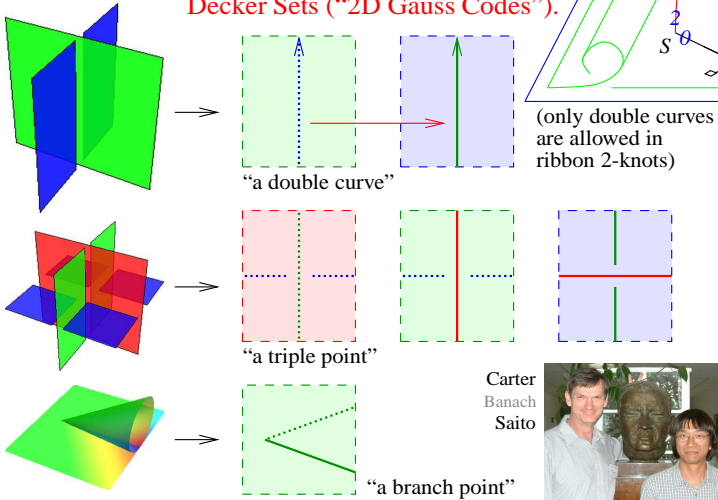
With $\kappa: (S = \mathbb{R}^2) \rightarrow M$, $\beta \in \Omega^0(S, \mathfrak{g})$, $\alpha \in \Omega^1(S, \mathfrak{g}^*)$, set

$$O(A, B, \kappa) := \int \mathcal{D}\beta \mathcal{D}\alpha \exp\left(\frac{i}{\hbar} \int_S \langle \beta, d_{\kappa^* A} \alpha + \kappa^* B \rangle\right).$$

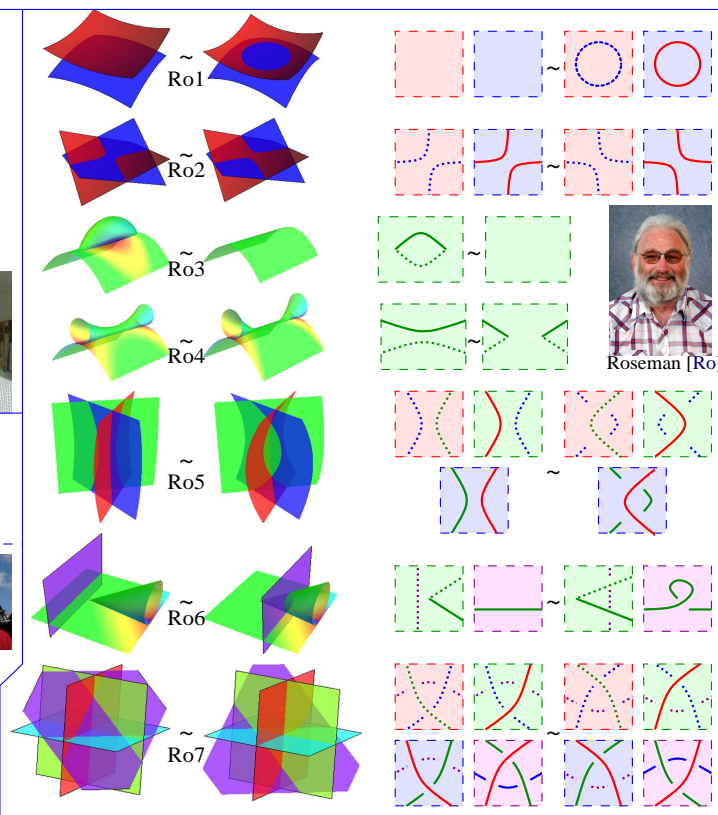
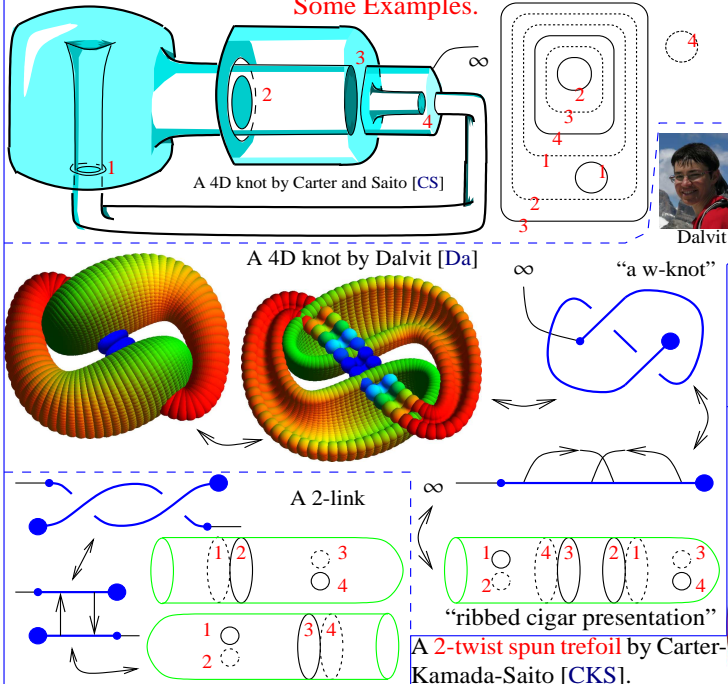
A BF Feynman Diagram.



Decker Sets ("2D Gauss Codes").



Some Examples.



Roseman [Ro]

