From V to F to KV following [AT].

```
logF = Λ[V]⟦1⟧ // dσ[{x, y} → {y, x}];
logF // EulerE // adSeries[(e^ad-1)/ad, logF, tb]
```

```
⟨x → LS[y/2, xy/6, 1/24 xyy, -1/180 xxxy + 1/80 xxyy + 1/360 xyyy,
  -1/720 xxxyy + 1/240 xxyyy + 1/240 xyxyy + 1/720 xxyxy -
  xyyyy/1440, xxxxy/5040 - xxxxyy/1344 + 13 xxxyyy/15120 + 1/840 xxyxyy +
  xxxyxy/3360 + xxyyyy/6720 + xyxyyy/1260 + xxyyxy/1680 - xyyyyy/10080, ...],
 y → LS[0, xy/12, 1/24 xyy, -1/360 xxxy + 1/120 xxyy + 1/180 xyyy,
  -1/720 xxxyy + 1/240 xxyyy + 1/240 xyxyy + 1/720 xxyxy -
  xyyyy/1440, xxxxy/10080 - xxxxyy/2016 + xxxyyy/1890 + xxyxyy/1120 + xxxyxy/5040 +
  xxyyyy/2520 + 1/840 xyxyyy + xxyyxy/1260 - xyyyyy/5040, ...]⟩
```

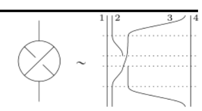
Solving for an associator Φ.

```
Φs[2, 1] = Φs[3, 1] = Φs[3, 2] = 0;
Φs[3, 1, 2] = 1/24; Φ = DKS[3, Φs];
SeriesSolve[Φ,
  (Φ^σ[3,2,1] ≡ -Φ) ∧
   (Φ ** Φ^σ[1,23,4] ** Φ^σ[2,3,4] ≡ Φ^σ[12,3,4] ** Φ^σ[1,2,34])];
Φ (* Can raise degree to 10 *)
```

SeriesSolve::ArbitrarilySetting : In degree 3 arbitrarily setting {Φs[3, 1, 1, 2] → 0}.
SeriesSolve::ArbitrarilySetting : In degree 5 arbitrarily setting {Φs[3, 1, 1, 1, 1, 2] → 0}.

```
DKS[0, 1/24 t13 t23, 0, -7 t13t23t23t23/5760 + 7 t13t13t23t23/5760 - t13t13t13t23/1440,
 0, 31 t13t23t23t23t23t23/967680 - 157 t13t13t23t23t13t23/1935360 -
 31 t13t23t13t23t23t23/387072 - 31 t13t13t23t23t23t23/483840 +
 11 t13t13t13t23t13t23/290304 + 31 t13t13t23t13t23t23/725760 + 83 t13t13t13t23t23t23/967680 -
 13 t13t13t13t13t23t23/241920 + t13t13t13t13t13t23/60480, ...]
```

The "buckle" Z_B, from Φ.



```
R = DKS[t[1, 2]/2];
ZB = (-Φ)^σ[13,2,4] ** Φ^σ[1,3,2] ** R^σ[2,3] ** (-Φ)^σ[1,2,3] **
   Φ^σ[12,3,4];
ZB@{4}
```

```
DKS[t23/2, -1/12 t13t23 - 1/24 t14t24 + 1/24 t14t34 + 1/12 t24t34,
 0, t13t23t23t23/5760 + 7 t14t24t24t24/5760 + t14t34t24t24/1920 -
 t14t34t34t24/1920 - 7 t14t34t34t34/5760 - t24t34t34t34/5760 + t14t24t34t24/1920 +
 t14t24t14t34/1920 - t14t34t24t34/1920 - 1/720 t13t13t23t23 +
 1/720 t13t13t13t23 - 7 t14t14t24t24/5760 + 7 t14t14t34t34/5760 -
 t14t24t34t34/5760 + t14t14t14t24/1440 - t14t14t14t34/1440 - 1/960 t14t14t24t34 +
 t14t24t24t34/5760 - 1/960 t24t24t34t34 - t24t24t24t34/5760, ...]
```

V from Z_B, following [AET, BND].

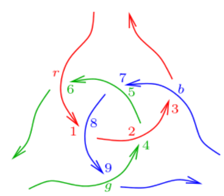
```
(El[ZB // αMap[1, 2, 3, 4], CWS[0]] // Γ // tη^1 // tη^3 //
   hη^2 // hη^4 // hσ[{3} → {2}] // tσ[{2, 4} → {1, 2}])⟦
 1⟧
```

```
⟨1 → LS[0, -12/24, 0, 71112/5760 - 71122/5760 + 1222/1440, 0,
  -3111112/967680 + 3111122/483840 - 831111222/967680 - 3111122/725760 - 3111112/645120 +
  1311222 22/241920 + 101 12 1222/1451520 + 527 1 122 12/5806080 - 122222/60480, ...],
 2 → LS[1/2, -12/12, 0, 1112/5760 - 1/720 1122 + 1/720 1222,
  -11112/7680 + 11122/3840 - 11212/6912,
  -111112/645120 + 23111122/483840 - 1311 1222/161280 - 112122/22680 - 41 11212/580608 +
  11222 2/15120 + 121222/12096 + 71 1122 12/483840 - 122222/30240, ...]⟩
```

The Borromean tangle.

```
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
iRs[a_, b_] := Es[⟨a → LS[0], b → -LS[LW@a]⟩, CWS[0]];
ξ = iRs[r, 6] Rs[2, 4] iRs[g, 9] Rs[5, 7] iRs[b, 3] Rs[8, 1];
```

```
Do[ξ = ξ // dm[r, k, r], {k, 1, 3}];
Do[ξ = ξ // dm[g, k, g], {k, 4, 6}];
Do[ξ = ξ // dm[b, k, b], {k, 7, 9}];
{ξ⟦1⟧r@{5}, ξ⟦2⟧@{5}} // Print
```



;

```
{LS[0, bg, 1/2 bbg + bgr + 1/2 bgg,
  1/6 bbbg + 1/2 bbgr + 1/2 bggr + 1/4 bbgg + 1/2 bgrr + 1/6 bggg,
  1/24 bbbbg + 1/6 bbbgr + 1/4 bbggr + 1/12 bbbgg +
  1/4 bbgrr + 1/6 bgggr + 1/4 bggrr - bbgrg +
  1/12 bbggg - 2 bbrgg + 1/6 bgrrr + 1/2 bgbgr -
  bgbrg - 1/12 bbgbg - 1/2 bgrgr + 1/24 bgggg, ...],
 CWS[0, 0, 2 bgr, bbgr - bgbr + bggr - bgrg + bgrr - brgr, bbbgr/3 -
  bbgbr/2 + bbggr/2 + bbgrg/2 + bbgrr/2 + bbrbg/2 - 3 bbrgr/2 + bgbrr/2 - 3 bggbr/2 + bgggr/3 -
  bggrg/2 + bggrr/2 + bgrgg/2 - 3 bgrrg/2 + bgrrr/3 + brggr/2 - brgrr/2 + brrgr/2, ...]}
```

References.

[AT] A. Alekseev and C. Torossian, *The Kashiwara-Vergne conjecture and Drinfeld's associators,* Annals of Mathematics **175** (2012) 415–463, arXiv:0802.4300.

[AET] A. Alekseev, B. Enriquez, and C. Torossian, *Drinfeld's associators, braid groups and an explicit solution of the Kashiwara-Vergne equations,* Publications Mathématiques de L'IHÉS, **112-1** (2010) 143–189, arXiv:0903.4067

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects I-IV,* ωεβ/WKO1, ωεβ/WKO2, ωεβ/WKO3, ωεβ/WKO4, and arXiv:1405.1956, arXiv:1405.1955, arXiv: not.yet×2.

Warning. Fidgety!

Video and more at http://www.math.toronto.edu/~drorbn/Talks/LesDiablerets-1508/